

## An Integrated Mathematical Framework for Optimizing Quay and Yard Design in Container Terminals

Mai-Ha Phan<sup>1,2\*</sup>, Linh Y Thai<sup>1,2</sup>, Hung Vu Manh<sup>1,2</sup>

<sup>1</sup>Department of Industrial System Engineering, Ho Chi Minh City University of Technology (HCMUT), 268 Ly Thuong Kiet Street, District 10, Ho Chi Minh City, Vietnam

<sup>2</sup>Vietnam National University Ho Chi Minh City, Linh Trung Ward, Thu Duc City, Vietnam  
*ptmaiha@hcmut.edu.vn (Corresponding author)*

**Abstract:** This paper introduces a comprehensive mathematical framework for facilitating the preliminary design of container terminals, with a particular focus on optimizing the quay and container yard layouts. The framework integrates technical and economic considerations crucial for terminal design, accommodating various input parameters, calculation algorithms, and output specifications tailored to the diverse operational requirements of each terminal segment. By incorporating cost considerations relevant to both the design and construction phases, the proposed model offers a holistic approach to optimizing container terminal layouts, enabling stakeholders to evaluate financial implications and make informed decisions amidst the evolving dynamics of global trade and shipping industry expansion. The framework leverages empirical formulas and adapts existing models to account for the specific constraints and requirements of container terminal design, providing a robust foundation for enhancing efficiency, sustainability, and cost-effectiveness throughout the operational spectrum.

**Keywords:** Container terminals, design model, mathematical model.

## 1. Introduction

Container terminals are pivotal in global trade and logistics networks, ensuring smooth goods transportation, fostering trade flows, and sustaining economic growth. Suboptimal designs can lead to inefficient space usage and operational bottlenecks, underscoring the need for continual optimization (Dávid, 2019). Designing these terminals involves intricate technical and economic considerations across storage yards, quays, and gates, necessitating extensive parameters and calculations (Lee and Kim, 2013).

Research efforts have focused on optimizing quay productivity, yard layout efficiency, and gate operations (Minh and Huynh, 2014, 2017). These aspects are crucial for maximizing throughput and minimizing turnaround times within container terminals. Despite the recognition of the need for continual optimization and the advancements, made in optimizing individual components of container terminals, there remains a critical gap in the, development of comprehensive mathematical models that can streamline the design process and facilitate the, integration of various components, such as the quay, container yard, and gate operations. While previous studies have made valuable contributions to optimizing individual components such as quay and yard designs, there remains a notable gap in addressing gate operations and integrating these components into a comprehensive framework. This gap hinders the development of holistic solutions that can optimize the overall efficiency and operational flow of container terminals. Closing this gap requires integrated approaches that encompass all aspects of terminal design, ensuring efficiency and sustainability throughout the entire operational spectrum.

This article proposes a mathematical framework to enhance the initial design phase of container ports. Leveraging empirical formulas, it facilitates parameter determination for quays, two types of yard layouts, and gates, while considering equipment and construction costs. By integrating these aspects, the model offers a robust approach to optimize container port designs and evaluate financial implications.

## 2. Literature Review on Factors Affecting Container Port Design

### 2.1. Literature review

Literature review result of factors affecting container port design presented in Table 1.

Table 1. Factors affecting container port design

Factor Research	Investment	Cargo	Vessels	Location	Equipment	Stakeholders	Permits	Port security	Customs facilities
Lee and Kim (2013)	x	x		x		x			x
Minh and Huynh (2014)			x			x	x		
Sharif (2011)	x		x	x	x	x		x	
Steenken et al. (2004)		x		x		x		x	x
Böse (2011)		x		x	x		x		
Liu et al. (2004)	x		x		x	x		x	
Carlo et al. (2014)	x	x	x	x		x		x	
Gou et al. (2020)	x	x			x	x	x	x	

Lee and Kim (2010)		x	x	x		x		x	x
Wiese et al. (2009)	x	x	x	x		x	x		x

(Source: Authors, 2024)

According to the literature review result, there are many factors affect container terminal design, as the following.

*Investment.* The source of investment determines the mission and scope of a container terminal project. Two main options include government funding and private funding. Government funding relies primarily on tax revenue as the main source of state income. However, without leveraging the strengths of non-state economic sectors, it becomes challenging to manage the financial aspects of the business and recoup the investment capital. On the other hand, private investment plays a crucial role in the establishment and operation of terminals. These ports are financed and managed with the primary goal of generating revenue.

Terminal investments require substantial capital, have long project life cycles, extended payback periods, and involve high risks. Especially for large-scale newly constructed terminals, it is necessary to make synchronous investments in both public and port infrastructure. Therefore, when considering investing in a container terminal, private investors will carefully evaluate and request specific commitments from contracting partners. These may include asking the government to establish conditions that restrict the licensing of new port projects in the same area, investing in traffic infrastructure, and dredging waterways. This conclusion is drawn on the basis of studies such as Lee and Kim (2013), Sharif (2011), Liu et al. (2004), Carlo et al. (2014), Gou et al. (2020), Wiese et al. (2009).

*Cargo.* The type of cargo being handled plays a crucial role in determining the primary functions, transportation modes, and associated features required for a container terminal. In container terminal design, the primary cargo object is the intermodal container. Containers are typically categorized into two common types: 20-foot and 40-foot containers. Additionally, in the United States and Canada, 53-foot containers have been introduced and are primarily utilized for domestic road and rail transportation. This conclusion is drawn on the basis of studies such as Lee and Kim, Steenken et al. (2004), Böse (2011), Carlo et al. (2014), Gou et al. (2020), Lee and Kim, (2010), Wiese et al. (2009).

*Vessels.* The type of vessel, its dimensions, and capacity have a significant influence on the required capacity of a container terminal. These vessel parameters also impact various aspects of terminal design, including the design of the quay, selection of cargo handling equipment, storage requirements, and modes of inland transportation.

The interaction between vessel characteristics and terminal characteristics can be observed in the following ways:

- Main Dimensions. Length - the factor that determines the width and curvature of the canal, the required size of the quay, and the maximum number of berths. Breadth and air draft - affect the choice of handling equipment and the width of the canal. Draft - determines the required depth along the berth.
- Cargo capacity: controls the minimum storage requirements for cargo vessels and can influence the loading and unloading processes, often determining the number of cranes needed to service each vessel.
- Designed vessel function: considers whether the vessel is equipped with cargo handling equipment or relies on external handling equipment. Container ships typically require external handling equipment rather than being self-equipped.
- Vessel Routine: considers the intermodal capacity requirements for import/export services and transshipment services, which can vary.

When selecting vessels, it is crucial to consider the development of container shipping. Misjudging the trend of ship size development can result in performance loss and low sustainability for the terminal. This conclusion is drawn on the basis of studies such as Minh and Huynh (2014), Sharif (2011), Liu et al. (2004), Carlo et al. (2014), Lee and Kim (2010), Wiese et al. (2009).

*Location.* The process of site selection for a container port involves several stages, starting from data collection and concluding with obtaining a government license. This selection process adheres to a triple-bottom-line philosophy, considering various factors such as waterside access, natural conditions, connections between different modes of transport, and the interests of stakeholders. When considering site selection for container ports, the following factors should be carefully evaluated:

- Water depth;
- Environmental conditions including meteorological and marine conditions;
- Availability of land;
- Good linkages in domestic transport or multimodal connections;
- Land conditions.

This conclusion is drawn on the basis of studies such as Lee and Kim (2013), Sharif (2011), Steenken et al. (2004), Böse (2011), Carlo et al. (2014), Lee and Kim (2010), Wiese et al. (2009).

*Equipment.* The process of reviewing and evaluating equipment is crucial in the planning, procurement, and construction of terminal facilities to ensure the efficient functioning of a container terminal. Various types of equipment are commonly considered during the design phase, including gantry cranes such as Rail Mounted Quay Cranes (RMQC) or Ship to Shore (STS) cranes, as well as intermodal container transport vehicles used in storage areas, such as Reach Stackers, Tractor-Trailer Units (TTU), and trucks. When designing a container terminal, specific specifications for cranes and intermodal freight vehicles should be considered. These specifications may include:

- The number of cranes and vehicles required;
- Size limits;
- Capacity;
- The ability to load and unload;
- Loading speed;
- Investment and operation cost;
- Other constraints on the working environment.

Equipment deployments will be designed with the primary task of creating enough cargo transported to balance the flow of goods. The queuing theory will be applied to find the right quantity and quality of equipment. This conclusion is drawn on the basis of studies such as Sharif (2011), Böse (2011), Liu et al. (2004), Gou et al. (2020).

*Stakeholders:* Various stakeholders play a role in the design process, including port managers, local municipalities, provincial and national governments, residents, potential operators, and environmentalists. Their interests and input must be taken into account to ensure a comprehensive and balanced design. This conclusion is drawn on the basis of studies such as Lee and Kim (2013), Minh and Huynh (2014), Sharif (2011), Steenken et al. (2004), Liu et al. (2004), Carlo et al. (2014), Gou et al. (2020), Lee and Kim (2010), Wiese et al. (2009).

*Permits:* Obtaining necessary permits is a crucial aspect of the design process. Port permits typically include clean air permits, construction permits, discharge permits, dredging permits, and water discharge permits. Adhering to regulatory requirements is essential for environmental compliance and sustainability. This conclusion is drawn on the basis of studies such as Minh and Huynh (2014), Böse (2011), Gou et al. (2020), Wiese et al. (2009).

*Port security:* Port security encompasses multiple aspects, including cargo security, port facility security, staff security, and maritime zone security. Effective port security measures involve collaboration between coastal authorities, customs, and border guards. Ports need to develop and implement a robust port security plan to safeguard against potential threats. This conclusion is drawn

on the basis of studies such as Sharif (2011), Steenken et al. (2004), Liu et al. (2004), Carlo et al. (2014), Gou et al. (2020), Lee and Kim (2010).

*Customs facilities:* Customs facilities are an integral part of a container terminal's design. Physical offices are required within warehouses and at border gates. These facilities support customs operations, including the detection of harmful agricultural products, smuggling (such as drugs and illicit funds), and the inspection of containers for radiation or stolen goods. This conclusion is drawn on the basis of studies such as Lee and Kim (2013), Steenken et al. (2004), Lee and Kim (2010), Wiese et al. (2009).

Considering these factors during the design process ensures that container terminals are designed to meet the needs and expectations of stakeholders, comply with regulatory requirements, ensure port security, and facilitate efficient customs operations.

## 2.2. Research gap

The proposed research aims to address a significant research gap in the field of container terminal design. While existing studies have explored various aspects of terminal operations, there remains a lack of comprehensive mathematical frameworks specifically tailored to facilitate the preliminary design phase of container terminals.

Existing research efforts have largely focused on optimizing specific components of terminal operations, such as quay productivity, yard layout efficiency, and gate operations. However, these studies often lack integration and fail to provide a holistic approach to terminal design. Furthermore, the current literature lacks detailed consideration of cost implications throughout the design and construction phases.

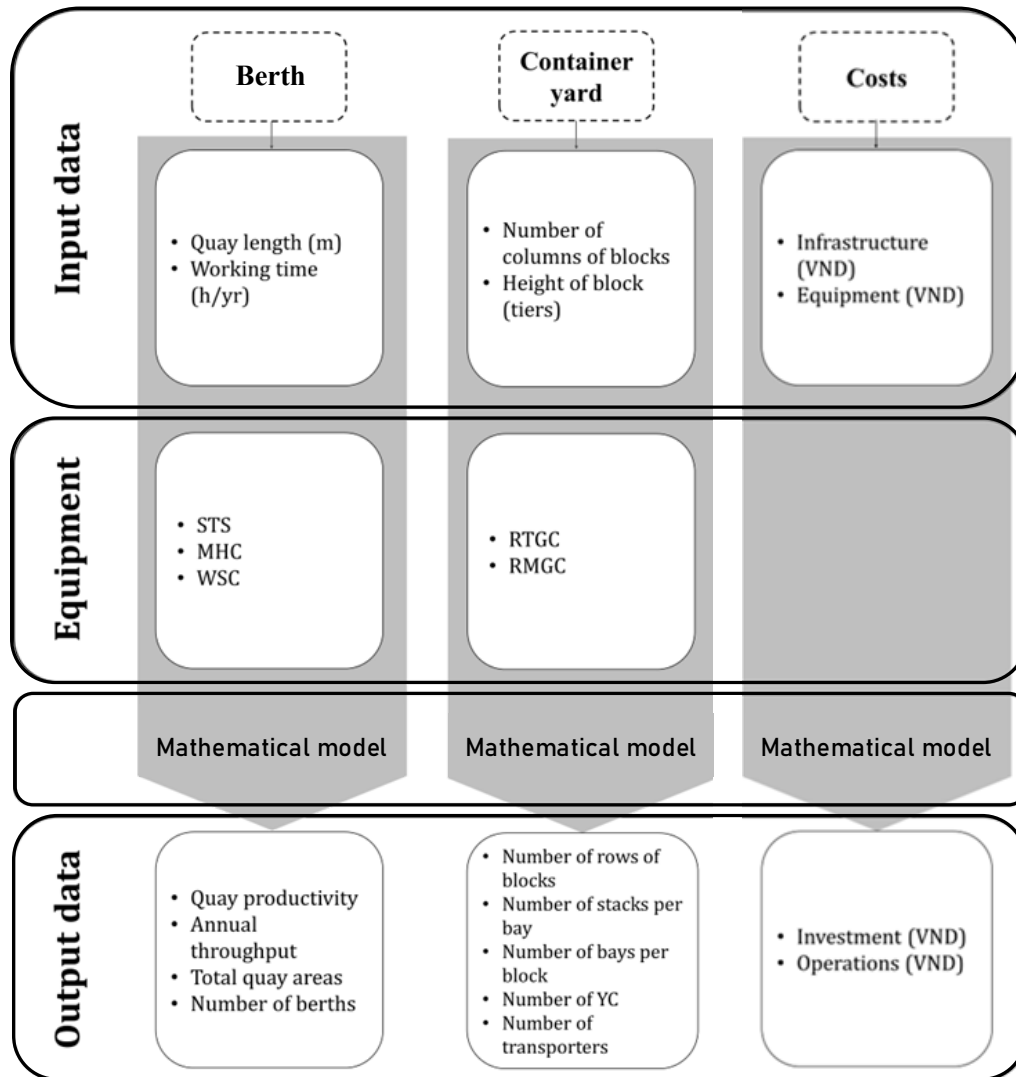
The necessity for efficient container port design has become increasingly paramount amidst the backdrop of consistent global trade growth and the economic turbulence induced by the Covid-19 pandemic. As container terminals play a crucial role in global trade and logistics networks, the development of a comprehensive mathematical framework for terminal design is essential to ensure operational efficiency, sustainability, and cost-effectiveness.

Therefore, this research seeks to fill this gap by proposing a novel mathematical framework that integrates diverse technical and economic considerations essential for terminal design.

## 3. Research methodology

As discussed in the previous section, the design of a container terminal involves considering a wide range of information and is an iterative process due to the need for compatibility among various subsystems. This iterative nature of the design process means that making changes or incorporating new ideas can be time-consuming, as it requires recalculating and redesigning different elements.

To address this challenge, a model is proposed to support the preliminary design of container terminals, aiming to save time and resources. This tool serves as a valuable aid in the design process, offering a streamlined approach to input and output data. Fig. 1 illustrates the input and output data involved in this computational template, which assists in expediting the design process and reducing the need for manual calculations and redesign efforts.



(Source: Authors proposal, 2024)

Fig. 1. Input data and output data of proposed model

Once all the necessary input data has been gathered and the appropriate equipment or technology has been selected for the designated areas, the calculation tool generates output data that provides the required information for designing each respective area. Additionally, the tool calculates investment costs and operational costs of the container terminal, enabling design engineers to evaluate the economic efficiency of terminal construction and assisting investors in making informed investment decisions.

The development of the proposed mathematical model for container terminal design follows a systematic approach, consisting of the following key steps:

1. Comprehensively understanding the components and operations of container terminals.
2. Analyzing the terminal design process and identifying the critical modules and areas.
3. Identifying and classifying the relevant input data.
4. Conducting a thorough review of existing empirical formulations and mathematical models, selecting appropriate models, and adapting them to suit the specific requirements of the study.
5. Defining the necessary output parameters for the design process.

## 4. Mathematical model

In this section, a computational model is introduced for the two primary subsystems of a seaport: the berth and container yard. Within each subsystem, a detailed analysis is conducted to identify the factors that impact their design, along with the commonly used types of equipment. Subsequently, a calculation model is presented to assist in the design process. Towards the end of the section, the costs associated with designing these subsystems are synthesized to create a comprehensive tool for preliminary design purposes.

### 4.1. Berth area

The berth design process typically follows one of two approaches:

- Determining berth length and quantity based on throughput demand.
- Calculating throughput based on given berth length and quantity, commonly used in terminals with limited space, such as in Asia.

Approach (1) is prevalent in European terminal studies where space is not a constraint, while approach (2) is favored in Asian terminals due to limited port sizes.

This discussion adopts approach (2) and assumes that each berth serves only one vessel at a time. This assumption aids in determining the required number of quay cranes for the entire berth based on the allocation of cranes per vessel and the total number of berths.

The quay length represents a significant and costly infrastructure investment, particularly in regions with high tidal ranges or water depths. It is crucial for estimating the number of berths and determining necessary equipment. Input parameters for calculating design requirements at the berth are detailed in Table 2, while Table 3 outlines corresponding output parameters.

Table 2. Input parameters for berth layout calculation

Symbol	Input parameters	Unit
$l_q$	length of the quay	m
$t_o$	(un)loading time	hrs/vessel
$t_m$	(un)mooring time	hrs/vessel
$\theta$	peak ratio for qc operations, the factor takes into account the highest level of operation of the terminal or qc ( $0 < \theta < 1$ ).	%
$D_p$	berth downtime factor, for berth maintenance and repair activities.	%
$T_p$	working time per day	hrs/day
$N_{dw}$	working days per week	days/week
$S_p$	parcel size	teu
$n_c$	number of containers per lift	teu
$t_t$	theoretical time for one move	min
$u_{qc}$	crane utilization ( $0.65 < u_{qc} < 1$ ).	%
$c_t$	coefficient of transshipment, activities of waiting, opening and closing the hatch of vessel.	%
$f_t$	coefficient of downtime	%
$U_b$	berth occupancy, berth occupancy is the ratio of the time the vessel spends on the total time spent in that period; a high occupancy (>70%) indicates a potentially congested berth, while (<50%) indicates a low availability.	(%)
$N_{20}$	number of teus.	
$N_{40}$	number of feus.	

$l_v$	vesel length	m
$L_{ml}$	berthing gap	m
$w_q$	width of the quay	m
$w_{rs}$	rail span	m
$w_{as}$	width of the access space	m
$w_{br}$	back reach of qc	m
$w_{tl}$	traffic lane width	m
$w_{qc}$	width of a crane	m
$q_{sp}$	spacing between cranes	m

(Source: Steeken et al., 2004; Authors proposal, 2024)

Table 3. Output parameters for berth layout calculation

Symbol	Output parameters
$N_b$	Number of berths
$n_{qc}$	Number of QCs per vessel

(Source: Source: Steeken et al., 2004; Authors proposal, 2024)

Calculation formulas are used for berth design:

The formulas used in the quay design calculations are derived and adapted from the seminal works of Ligteringen (1999), Böse (2011), Dávid (2009), and the guidelines provided by PIANC (2014b). These formulas have been carefully selected and adjusted to account for the specific operational requirements and constraints inherent in the container terminal design process.

$\gamma$  = TEU factor. The ratio between 40ft and 20ft containers. The factor of 2.N40 means two 20ft containers equals one 40ft container. The value 1.6 represents 60% of the 40ft container (Ligteringen, 1999).

$$\gamma = \frac{N_{20} + 2 \cdot N_{40}}{N_{20} + N_{40}}$$

$p_{qc}$  = Quay crane productivity (TEU/hr). The net capacity of QC is calculated after dividing the number of TEUs hoisting and lowering once divided by the theoretical time of one time. This theoretical capacity is then multiplied by the usefulness and losses due to the waiting and movement of the crane. Since the theoretical time for hoisting/lowering of a crane is often expressed in terms of minutes, multiply 60 to convert it to hours (Ligteringen, 1999).

$$p_{qc} = 60 \frac{n_c}{t_t} u_{qc} c_t f_t$$

Length of the berth is calculated by multiplying the number of berths by the required length of one berth (the length of the vessel plus the extra distance for mooring). The factor 1.1 calculates the variation in the length of the train because  $l_v$  only denotes the mean length. Results from the study of UNCTAD (1984).

In terms of the extra mooring distance, when the vessel arrives at the berth, the vessel will anchor the front and rear of the vessel, so the length of the berth must be twice the length of the extra distance. However, in practice, there is no physical separation between berths, so the forward mooring distance can be shared for the rear mooring distance. Therefore, in the distance required formula for a berth only need to add the average length of one vessel for one of the extra distances and one of the extra lengths for the last vessel since it has no more ships (Böse, 2011).

$$N_b = \frac{l_q - L_{ml}}{(l_v + L_{ml}) \times 1.1}$$

$T_s$  = Total service time (hrs/vessel). Total service time of the vessel includes the loading and unloading time of vessel and time of mooring (Böse, 2011).

$$T_s = t_m + t_o$$

$T_b$  = Total working time per week (hrs/week). Working time per week is calculated by working time per day multiplied by the number of working days per week and berth downtime factor (Böse, 2011).

$$T_b = (1 - D_p) \times T_p \times N_{dw}$$

$L_{br}$  = Berth length requirement (hrs.m/week). Usually, the length of a berth required is calculated by adding the length of the vessel plus the distance it uses for mooring. However, in reality, the vessel not only occupies the berth physically in terms of distance (m) but also occupies in terms of the time the vessel is docked at the berth (hours) (Böse, 2011).

Based on research and reality, vessels often moor at berths for a total of up to a few days to less than a week, so the formula will calculate occupancy in terms of time per week (Dávid, 2009).

$$L_{br} = \frac{T_b \times U_b \times l_q}{\theta}$$

$N_v$  = Number of vessel arrival (vessels/week) (expected) (Dávid, 2009).

$$N_v = \frac{L_{br}}{(l_v + L_{ml}) \times T_s}$$

The number of quay cranes allocated to a vessel will depend on vessel capacity. By dividing the capacity of the vessel by the capacity of the crane then multiple by the loading and unloading time (Dávid, 2009).

$$n_{qc} = \frac{S_p}{p_{qc} \times t_o}$$

$w_{ar}$  = Access road width (m). The width of the internal truck driveway includes the 4-lane traffic road for internal vehicles multiplied by 1.5 the turn away factor (Dávid, 2009).

$$w_{ar} = 4 \times w_{tl} \times 1.5$$

$w_{apron}$  = Total apron width (m). The total apron area width is calculated by the sum of the width of the quay wall, access space, rail gauge, back reach of QC and internal truck traffic (Dávid, 2009).

$$w_{apron} = w_q + w_{as} + w_{rs} + w_{br} + w_{ar}$$

$S_q$  = Total quay area (m<sup>2</sup>). The total area of the quay area is equal to width of the apron area times length of the quay (Dávid, 2009).

$$S_q = w_{apron} \times l_q$$

$Q_c$  = Quay productivity (TEU/day). The capacity of the berth is the number of berths multiplied by the working time of berth multiplied by the capacity of a berth (where the capacity of the QC is multiplied by the number of QC), plus the TEU factor (PIANC, 2014b).

$$Q_c = N_b \times T_p \times n_{qc} \times p_{qc} \times \gamma$$

$N_{tqc}$  = Total number of QCs. Total QC of the berth, here because assuming a berth only serves one vessel at a time so can multiply the QC number assigned to one vessel by the number of berths (PIANC, 2014b).

$$N_{tqc} = N_b \times n_{qc}$$

$AT$  = Annual throughput (TEU/yr). Calculated by the capacity of the berth per day multiplied by the number of working days per year, plus berth downtime factor (PIANC, 2014b).

$$AT = Q_c \times N_{dw} \times (1 - D_p) \times 52$$

$l_{nc}$  = Length of total QC (m). The occupied QC length (crane width and distance between two QCs) multiplied by the total number of QC in the quay plus extra distance of the final QC (PIANC, 2014b).

$$l_{nc} = (q_{sp} + w_{qc}) * N_{tqc} + q_{sp}$$

## 4.2. Container yard area

The assumptions and computational formulas for the parallel yard layouts are derived and adjusted based on the research of Lee and Kim (2010, 2013). This layout type involves YCs moving between blocks while trucks and transporters navigate along aisles to transport containers. Assumptions include rectangular terminal shape, uniform YC distribution, negligible YC intersection, proportional deployment of YCs based on activity type, segregation of blocks for inbound and loading containers, uniform block sizes, fixed quay length determining yard width, gate placement, single-cycle container delivery by internal trucks, and one-container transport per internal truck. The layout illustration features 3 columns, 5 rows of blocks, 6 stacks per bay, and 6 YCs per row. The following notation conventions are mainly used for the YC cycle time calculation model (Table 4).

Table 4. Notations of YC cycle time calculation model

Symbol	Notation
$T_{zw}^{xy}$	handling time, which is a random variable, for the movement of a YC whose starting position, ending position, type of motion, and state of equipment (loaded or empty) are, respectively, $x, y, z,$ and $w$ (min).
$R_{WH}$	number of re-handles required to pick up a random container from a bay with $W$ rows and $H$ tiers; this is a random variable
$t_{zw}^{xy}$	" $T$ " in $T_{zw}^{xy}$ is replaced with " $t$ " when the travel distance is a constant rather than a random variable (min).
<i>Type of motion—subscripts</i>	
$g$	gantry travel
$t$	trolley travel
$h$	hoisting up or down
$r$	releasing a container
$p$	picking up a container
$R$	re-handling operation
<i>State of equipment—subscripts</i>	
$e$	empty movement
$l$	loaded movement
<i>Starting and ending positions—superscripts</i>	
$a$	an arbitrary (random) position within the range in which a piece of handling equipment moves for picking up or releasing a container from or into the storage area
$b$	boundary of a container stacking block
$c$	position of the spreader that is picking up or releasing a container from or onto a chassis

(Source: Lee and Kim, 2010, 2013; Authors proposal, 2024)

Table 5 illustrates input parameters of container yard layout calculation model. Decision variables are also shown as Table 6 below.

Table 5. Input parameters of parallel container yard calculation model

Symbol	Input parameter
$l_b$	length of a bay (m)
$w_r$	width of a stack (m)
$w_h$	width of a horizontal aisle between adjacent blocks in the layout including the width of a lane for driving (m)
$w_v$	width of a vertical aisle between adjacent blocks in the layout including the width of a lane for driving (m)
$n_r$	number of containers outbound containers during a year
$n_t$	number of transshipment containers during a year

Symbol	Input parameter
$n_d$	number of inbound containers during a year
$d_r$	average dwell time (in working time) of outbound containers at the yard (hrs)
$d_t$	average dwell time (in working time) of transshipment containers at the yard (hrs)
$d_d$	average dwell time (in working time) of inbound containers at the yard (hrs)
$h$	total working time per year (hrs)
$n_{YC}$	number of ycs installed at each row of blocks in the layout
$u$	average number of QCs allocated to a vessel
$\delta$	peak ratio for arriving containers by road trucks ( $0 < \delta < 1$ )
$\omega$	average throughput rate at the quay per hour
$\lambda$	arrival rate of road trucks for receiving and delivery containers, incorporating the peak arrivals, per hour, which can be calculated by $(1 + \delta)\{(n_r + n_d)/h\}$
$s_o$	average storage space requirement (TEU) for loading containers, which can be evaluated as $\gamma(n_r d_r + n_t d_t)/(uh)$
$s_I$	average storage space requirement (TEU) for inbound containers, which can be represented by $\gamma(n_d d_d)/(uh)$
$t_r$	maximum allowed average turnaround time of road trucks for a outbound container (hrs)
$t_d$	maximum allowed average turnaround time of road trucks for a inbound container (hrs)
$t_l$	maximum allowed average system time of transporters at a block for a loading container (hrs)
$t_u$	maximum allowed average system time of transporters at a block for a discharging container (hrs)
$v_e$	speed of empty travel of a vehicle (m/hr)
$v_l$	speed of loaded travel of a vehicle (m/hr)
$f_G$	construction cost of the ground space equivalent to a square meter, which is converted to the equivalent annual cost; this includes the investment capital cost for the land and the construction of the ground
$f_{YC}$	fixed overhead cost of a YC per year. this is related to the investment capital cost for purchasing a YC
$c_{YC}$	operating cost per minute of a YC including labor, fuel, maintenance, and overhead costs. this explains the cost term which increases as the operation time of YCs increases
$f_{TR}$	fixed overhead cost of a transporter per minute
$c_{TR}$	operating cost per minute of a transporter including labor, fuel, maintenance and overhead costs
$\rho$	traffic density is usually calculated as the average arrival rate trucks times the expected cycle time of YC

(Source: Lee and Kim, 2010, 2013; Authors proposal, 2024)

Table 6. Decision variables of parallel container yard calculation model

Symbol	Decision variable
$N$	number of columns of blocks in the layout
$H$	maximum height of stacks
$W$	number of stacks in a bay

(Source: Lee and Kim, 2010, 2013; Authors proposal, 2024)

### Dependent variables (Source: Lee and Kim, 2010, 2013; Authors proposal, 2024)

$S_{par}$  = Total parallel area.

$$S_{par} = l_q[(w_r W + w_h)R + w_h]$$

$R$  = Number of rows of blocks in the layout, calculated by taking the total storage space required for the inbound and outbound containers divided by the total number of slots to find out the number of rows of blocks in the layout.

$$R = \frac{S_o + S_l}{NBHW}$$

$B$  = Number of bays per block in the layout. The number of bays  $B$  in each block is calculated by dividing length of one block by length of a bay  $l_b$  (equivalent to the length of a container).

$$B = \frac{l_q - (N + 1)w_v}{l_b N}$$

$H_r$  = Average height of stacks in the number of tiers.

$$H_r = Hu$$

$A_h^{YC}$  = Arrival rate of road trucks from the hinterland for receiving and delivery containers, incorporating the peak arrivals, per hour per YC.

$$A_h^{YC} = \lambda / (n_{YC} R \frac{n_r + 2n_t + n_d}{2(n_r + n_t + n_d)})$$

$A_s^{YC}$  = Arrival rate of transporters for loading and discharging containers, incorporating the peak arrivals, per hour per YC.

$$A_s^{YC} = \omega(1 + \theta) / (n_{YC} R \frac{n_r + n_d}{2(n_r + n_t + n_d)})$$

**Calculation formulas are used for parallel yard layouts design (Source: Lee and Kim, 2010, 2013; Authors proposal, 2024)**

$f_{YC}(H, W)$  = The fixed cost of YC per year, depends on height and width of the block.

$$f_{YC} = \alpha + \beta(W + 2H)$$

The formula for YC fixed cost is provided by a YC manufacturer in Korea, depending on the size of YC. Note that  $\alpha$  represents a fixed cost, while  $\beta(W + 2H)$  represents the material and manufacturing cost for the crane which is proportional to the sum of the bay width and twice of the height of stacks.

The comprehensive expressions for the expected cycle times and variances of different types of operations, considering the speeds of gantry travel, trolley travel, and hoisting/lowering movements, can be found in Lee and Kim's work (2010). Additionally, the detailed expressions for the expected waiting times associated with various operations can be found in Lee and Kim's study (2010a).

There are two types of movements of YC:

1. Rectangular movement: This assumes that YCs move in a rectangular manner, which means that the gantry, trolley, and vertical (hoisting/lowering) movements are not done simultaneously. Handling sequences for various types of operation are provided together with explanations of derivation of estimators for the elements of handling.
2. Tchebychev movement: This assumes that the trolley of a YC moves while the YC travels in the gantry direction. This type of movement is known as Tchebychev travel (Francis et al., 1992). For safety reasons, however, the vertical (hoisting/lowering) movement cannot be done simultaneously with the gantry or trolley movement.

$C_r(H, W)$  = YC cycle time for a receiving operation during which a YC receives an outbound container from road trucks or a transshipment container from a transporter under a given combination of  $H$  and  $W$  (min).

- Rectangular movement:

$$E(C_r) = E(T_{ge}^{aa}) + E(T_{te}^{ac}) + t_{he}^{bc} + t_p + t_{hl}^{cb} + E(T_{tl}^{ca}) + E(T_{hl}^{ba}) + t_r + E(T_{he}^{ab})$$

- Tchebychev movement:

$$E(C_r) = E[\text{Max}(T_{te}^{ac}, T_{ge}^{aa})] + t_{he}^{bc} + t_p + t_{hl}^{cb} + E(T_{tl}^{ca}) + E(T_{hl}^{ba}) + t_r + E(T_{he}^{ab})$$

$C_l(H, W)$  = YC cycle time for a loading operation in which a YC transfers an outbound or a transshipment container to a transporter that delivers it to a QC for loading onto a vessel for a given combination of  $H$  and  $W$  (min)

- Rectangular movement:

$$E(C_l) = \frac{1}{l_0} E(T_{ge}^{aa}) + E(T_{te}^{ca}) + E(T_{he}^{ba}) + t_p + E(T_{hl}^{ab}) + E(T_{tl}^{ac}) + t_{hl}^{bc} + t_r + t_{he}^{cb}$$

- Tchebychev movement:

$$E(C_l) = E[\text{Max}(T_{te}^{ca}, T_{ge}^{aa})] + \frac{l_0 - 1}{l_0} E(T_{te}^{ca}) + E(T_{he}^{ba}) + t_p + E(T_{hl}^{ab}) + E(T_{tl}^{ac}) + t_{hl}^{bc} + t_r + t_{he}^{cb}$$

$C_u(H, W)$  = YC cycle time for a discharging operation in which a YC receives an inbound container, discharged from a vessel, from a transporter for a given combination of  $H$  and  $W$  (min).

- Rectangular movement:

$$E(C_u) = \frac{1}{s_{WH}} E(T_{ge}^{aa}) + E(T_{te}^{ac}) + t_{he}^{bc} + t_p + t_{hl}^{cb} + E(T_{tl}^{ca}) + E(T_{hl}^{ba}) + t_r + E(T_{he}^{ab})$$

- Tchebychev movement:

$$E(C_u) = E[\text{Max}(T_{te}^{ac}, T_{ge}^{aa})] + \frac{s_{WH} - 1}{s_{WH}} E(T_{te}^{ac}) + t_{he}^{bc} + t_p + t_{hl}^{cb} + E(T_{tl}^{ca}) + E(T_{hl}^{ba}) + t_r + E(T_{he}^{ab})$$

$C_d(H, H_r, W)$  = YC cycle time for a delivery operation in which a YC transfers an inbound container to a road truck for a given combination of  $H, H_r$ , and  $W$  (min).

- Rectangular movement:

$$E(C_d) = E(T_{ge}^{aa}) + E(T_{te}^{ca}) + E(T_R) + E(T_{he}^{ba}) + t_p + E(T_{hl}^{ab}) + E(T_{tl}^{ac}) + t_{hl}^{bc} + t_r + t_{he}^{cb}$$

- Tchebychev movement:

$$E(C_d) = E[\text{Max}(T_{te}^{ca}, T_{ge}^{aa})] + R_{WH} E(T_R) + E(T_{he}^{ba}) + t_p + E(T_{hl}^{ab}) + E(T_{tl}^{ac}) + t_{hl}^{bc} + t_r + t_{he}^{cb}$$

Waiting time of road trucks (internal trucks) for all operation of YC is calculated based on the formula for calculating the average Pollaczek-Khintchine (P-K) waiting time for an M/G/1 queuing system (Gross and Harris, 1998).

$W_r(R, H, W)$  = Waiting time of a road truck (transporter) for transferring an outbound (transshipment) container to a YC a given combination of  $R, H$ , and  $W$  (min).

$$W_r = \frac{\rho E(C_r)}{2(1 - \rho)} \left( 1 + \frac{\text{Var}^2(C_r)}{E^2(C_r)} \right)$$

$W_l(R, H, W)$  = Waiting time of a transporter for receiving a loading container from a YC for a given combination of  $R, H$ , and  $W$  (min).

$$W_l = \frac{\rho E(C_l)}{2(1 - \rho)} \left( 1 + \frac{\text{Var}^2(C_l)}{E^2(C_l)} \right)$$

$W_u(R, H, W)$  = Waiting time of a transporter for transferring an inbound container to a YC for a given combination of  $R, H$ , and  $W$  (min).

$$W_u = \frac{\rho E(C_u)}{2(1-\rho)} \left( 1 + \frac{Var^2(C_u)}{E^2(C_u)} \right)$$

$W_d(H, H_r, W)$  = Waiting time of a road truck for receiving an inbound container from a YC for a given combination of  $R, H$ , and  $W$  (min).

$$W_d = \frac{\rho E(C_d)}{2(1-\rho)} \left( 1 + \frac{Var^2(C_d)}{E^2(C_d)} \right)$$

$D_g(N, R, W)$  = Round-trip travel distance of road trucks between the gate and a random position in the yard for given values of  $N, R$ , and  $W$  (m).

$$E[D_g(N, R, W)] = \begin{cases} \frac{(N+1)^2}{2N^2} l_q + ((w_r W + w_h)R + w_h), & N \text{ is odd} \\ \frac{(N+2)}{2N} l_q + ((w_r W + w_h)R + w_h), & N \text{ is even} \end{cases}$$

$T_g(N, R, W)$  = Travel time corresponding to  $D_g(N, R, W)$ .

$$E[T_g(N, R, W)] = \frac{1}{2} E[D_g(N, R, W)] \left( \frac{1}{v_e} + \frac{1}{v_l} \right)$$

$D_q(N, R, W)$  = Round-trip travel distance of transporters between a random position of the quay and a random position in the yard for given values of  $N, R$ , and  $W$  (m).

$$E[D_q(N, R, W)] = \frac{(2N^2 + 3N + 1)}{3N^2} l_q + ((w_r W + w_h)R + w_h)$$

$T_q(N, R, W)$  = Travel time corresponding to  $D_q(N, R, W)$ .

$$E[T_q(N, R, W)] = \frac{1}{2} E[D_q(N, R, W)] \left( \frac{1}{v_e} + \frac{1}{v_l} \right)$$

### 4.3. Cost

#### 4.3.1. Quay design costs

Input parameters are presented in Table 10 to facilitate cost calculations for the quay and quay crane (QC).

Table 10. Input parameters of Quay costs calculation

Symbol	Input parameters
$c_{qw}$	construction cost of quay wall (VND/ m <sup>2</sup> )
$c_q$	construction cost of the ground space for quay (VND/ m <sup>2</sup> )
$c_{fu}$	cost for mooring infrastructure (VND), these infrastructures include the mooring poles, the protective gaskets fitted along the length of the berth
$c_{qc}$	the fixed overhead cost of QCs (VND)
$m_{qc}$	QC's one-hour operation cost (VND/hr), this cost may include maintenance costs, fuel used by qc.
$t_{sh}$	working hour per shift (hrs)
$l_{cb}$	labor cost for a gang per QC (VND/hr). QC staff is assigned to the ship. this includes port workers as well as the QC operator

(Source: Schonfeld and Sharafeldien, 1985; Authors proposal, 2024)

The calculation of investment and operation costs for the quay involves several components, including ground construction cost, equipment investment cost, QC operation cost, berth cost, and labor cost. The formulas used for these calculations are derived from the study conducted by Schonfeld, P., and Sharafeldien, O. (1985).

The construction cost of the ground space for quay:

$$FC_q = l_q \times (c_{qw} + c_{fu}) + S_q \times c_q$$

The fixed overhead cost of QCs:

$$FC_{qc} = N_{tqc} \times c_{qc}$$

Operating cost when handling 1 TEU:

$$OC_{qc} = \frac{m_{qc}}{p_{qc}}$$

The operating cost of quay:

$$OC_q = OC_{qc} \times AT$$

Labour cost:

$$C_{lb} = 52 \times N_v \times n_{qc} \times l \times l_{cb}$$

whereas  $l = \max(T_s, t_{sh})$

### 4.3.2. Container yard costs

#### Parallel yard layouts

The following shows how to calculate the investment and operating costs related to parallel layout, including: construction cost for the container yard, investment cost for YC, operating cost of YC, fixed cost operating cost of internal trucks (Schonfeld and Sharafeldien, 1985; Authors proposal, 2024).

The construction cost of the ground space for blocks and aisles in the layout:

$$l_q f_G((w_r W + w_h)R + w_h)$$

The fixed overhead cost of YCs in the layout:

$$n_{YC} R f_{YC}(H, W) = n_{YC} R[\alpha + \beta(W + 2H)]$$

The operating cost of YCs in the layout:

$$c_{YC}(n_r + n_t)\{E[C_r(H, W)] + E[C_l(H, W)]\} + c_{YC} n_d \{E[C_d(H, H_r, W)] + E[C_u(H, W)]\}$$

The total annual operation cost of transporters for outbound and transshipment containers:

$$c_{TR}(n_r + n_t)\{E[T_q(N, R, W)] + E[W_l(R, H, W)] + E[C_l(H, W)]\} + c_{TR} n_t \{E[T_q(N, R, W)] + E[W_r(R, H, W)] + E[C_r(H, W)]\}$$

The operating cost of transporters for inbound containers:

$$c_{TR} n_d \{E[T_q(N, R, W)] + E[W_u(R, H, W)] + E[C_u(H, W)]\}$$

The fixed overhead and operating cost of transporters:

$$\begin{aligned} & (\xi f_{TR} + c_{TR})(n_r + n_t)\{E[T_q(N, R, W)] + E[W_l(R, H, W)] + E[C_l(H, W)]\} \\ & + (\xi f_{TR} + c_{TR}) n_t \{E[T_q(N, R, W)] + E[W_r(R, H, W)] + E[C_r(H, W)]\} \\ & + (\xi f_{TR} + c_{TR}) n_d \{E[T_q(N, R, W)] + E[W_u(R, H, W)] + E[C_u(H, W)]\} \end{aligned}$$

where  $\xi$  represents the factor to convert the total expected operation time of transporters to the required number of transporters ( $0 < \xi < 1$ ).

#### Perpendicular yard layouts (Schonfeld and Sharafeldien, 1985; Authors proposal, 2024)

The construction cost of the ground space for blocks and aisles in the layout:

$$l_q f_G(l_b B + 2w_h)$$

The fixed overhead cost of YCs in the layout:

$$m_{YC} N f_{YC}(H, W) = m_{YC} N[\alpha + \beta(W + 2H)]$$

The fixed overhead and operating cost of transporters in the layout:

$$\begin{aligned} & (\xi f_{TR} + c_{TR})(n_r + n_t)\{t_{TR} + E[W_l(R, H, W)] + E[C_l(H, W)]\} \\ & + (\xi f_{TR} + c_{TR})n_t\{t_{TR} + E[W_r(R, H, W)] + E[C_r(H, W)]\} \\ & + (\xi f_{TR} + c_{TR})n_d\{t_{TR} + E[W_u(R, H, W)] + E[C_u(H, W)]\} \end{aligned}$$

## 5. Conclusion

This study presents a comprehensive mathematical framework that addresses a critical gap in the field of container terminal design by integrating the optimization of key components, including the quay and container yard layouts. The proposed model incorporates both technical and economic considerations, providing stakeholders with a valuable tool for evaluating the financial implications of design decisions and optimizing container terminal layouts to meet the evolving demands of global trade and shipping industry expansion.

The framework's ability to accommodate various input parameters, calculation algorithms, and output specifications tailored to the diverse operational requirements of each terminal segment represents a significant contribution to the field. By leveraging empirical formulas and adapting existing models to the specific constraints and requirements of container terminal design, the study offers a robust foundation for enhancing efficiency, sustainability, and cost-effectiveness throughout the operational spectrum.

While the research presented in this paper represents a significant advancement, there is potential for further refinement and extension. Future research could explore the integration of additional terminal components, such as gate operations, into the mathematical framework, further enhancing its comprehensiveness. Additionally, the development of user-friendly software tools or graphical user interfaces based on the proposed model could facilitate its practical implementation and adoption by industry stakeholders.

Overall, this study contributes to the body of knowledge in the field of container terminal design and optimization, providing a valuable resource for researchers, designers, and industry professionals seeking to develop efficient and cost-effective solutions in the face of evolving global trade dynamics and shipping industry expansion.

## Acknowledgment

This research is funded by Vietnam National University Ho Chi Minh city under grant number C2022-20-10. We acknowledge Ho Chi Minh City University of Technology (HCMUT), VNU-HCM for supporting this study.

## References

- Dávid, A. (2009). Kontajnerové prekladače Post Panamax. *Doprava a spoje*, 1(1), 1-6.
- Lee, B. K., & Kim, K. H. (2013). Optimizing the yard layout in container terminals. *OR spectrum*, 35(2), 363-398.
- Minh, C. C., & Huynh, N. (2014). Planning-level tool for assessing and optimizing gate layout for marine container terminals. *Transportation Research Record*, 2409(1), 31-39.
- Minh, C. C., & Huynh, N. (2017). Optimal design of container terminal gate layout. *International Journal of Shipping and Transport Logistics*, 9(5), 640-650.
- Sharif Mohseni, N. (2011). Developing a Tool for Designing a Container Terminal Yard.
- Steenken, D., Voß, S., & Stahlbock, R. (2004). Container terminal operation and operations research-a classification and literature review. *OR spectrum*, 26(1), 3-49.

- Ligteringen, Han & H.Velsink,. (1999). Ports and Terminals.
- Böse, J. W. (2011). *Handbook of terminal planning* (Vol. 49, p. 433). New York: Springer.
- Liu, C. I., Jula, H., Vukadinovic, K., & Ioannou, P. (2004). Automated guided vehicle system for two container yard layouts. *Transportation Research Part C: Emerging Technologies*, 12(5), 349-368.
- Petering, M. E. (2009). Effect of block width and storage yard layout on marine container terminal performance. *Transportation Research Part E: Logistics and Transportation Review*, 45(4), 591-610.
- Carlo, H. J., Vis, I. F., & Roodbergen, K. J. (2014). Storage yard operations in container terminals: Literature overview, trends, and research directions. *European journal of operational research*, 235(2), 412-430.
- Wiese, J., Kliewer, N., & Suhl, L. (2009). A survey of container terminal characteristics and equipment types. *Paderborn: Decision Support & Operations Research Lab, Paderborn University*.
- Kemme, N. (2012). Effects of storage block layout and automated yard crane systems on the performance of seaport container terminals. *OR spectrum*, 34(3), 563-591.
- Guo, P., Wang, L., Xue, C., & Wang, Y. (2020). Dispatching Rules for Scheduling Twin Automated Gantry Cranes in an Automated Railroad Container Terminal. *Arabian Journal for Science and Engineering*, 45(3), 2205-2217.
- Lee, B. K., & Kim, K. H. (2010). Optimizing the block size in container yards. *Transportation Research Part E: Logistics and Transportation Review*, 46(1), 120-135.
- Lee, B. K., & Kim, K. H. (2010). Comparison and evaluation of various cycle-time models for yard cranes in container terminals. *International Journal of Production Economics*, 126(2), 350-360.
- Francis, R. L., McGinnis, L. F., & White, J. A. (1992). *Facility layout and location: an analytical approach*. Pearson College Division.
- Guan, C. Q. (2009). Analysis of marine container terminal gate congestion, truck waiting cost, and system optimization.
- Schonfeld, P., & Sharafeldien, O. (1985). Optimal berth and crane combinations in containerports. *Journal of waterway, port, coastal, and ocean engineering*, 111(6), 1060-1072.