ISSN 2409-2665
Journal of Logistics, Informatics and Service Science
Vol. 11 (2024) No. 10, pp. 276-293
DOI:10.33168/JLISS.2024.1015

Customer Behavior Prediction using Machine Learning with Fuzzy Multiple Linear Regression

Barraq Subhi Kaml*, Ali Abdulhafidh Ibrahim
College of Business Economics, Al-Nahrain University, Iraq,
dr.barraq@ nahrainuniv.edu.iq (Corresponding Author), dr_ali9@yahoo.com

Abstract. Accurate prediction of customer behavior is crucial for businesses to optimize marketing strategies, product offerings, and customer relationships. Traditional regression methods often struggle when dealing with imprecise or fuzzy data, which is common in customer behavior datasets. This study proposes an integrated approach that combines the strengths of fuzzy multiple linear regression (FMLR) and machine learning techniques to enhance customer behavior prediction. FMLR is employed to handle the inherent fuzziness in customer data, while machine learning principles are leveraged for model training, optimization, and generalization. The proposed method is applied to a dataset containing customer spending scores along with age and income information. The results demonstrate the potential of this approach in capturing complex customer behavior patterns, enabling businesses to gain actionable insights for targeted marketing, cross-selling, and customer retention strategies. The study contributes to the field by introducing a robust and interpretable framework that harnesses the synergy between FMLR and machine learning for improved customer behavior predictions.

Keywords: Fuzzy Multiple Linear Regression, Fuzzy Least Squares Estimators, α – cut, Machine Learning, linear programming model, Customer Behavior.

1. Introduction

In the last years, the combining between Machine Learning and Fuzzy Multiple Linear Regression (FMLR) has proved to be an important approach in decode and predicting customer behavior (Ozde and Okan, 2018). With the increasing dependence on data-driven decision-making, businesses have turned to MLR, a sub of Machine Learning, to get great insights into consumer options, desire, and future activities (Raed, et al., 2023). This paper has a try to investigate the power transformation of Machine Learning, especially MLR, in the field of customer behavior prediction, contributions businesses a unique chance to resolve the complication of customer interactive and desire.

The convergence of advanced computational techniques with the deep of consumer data allows a model take in our comprehension of customer behavior (Feng and Joey, 2023).Influenced by MLR algorithms, we seek to create predictive models that go beyond conventional constraints. Grounded in machine learning principles, these models provide businesses with executable perception into customer preservation, Buying tendencies, and the strategic optimization of marketing. In this exploration, we illuminate between Machine Learning and MLR. We delve into the essential principles that underpin MLR's ability to dissect complex customer data, uncover hidden trends, and facilitate data-driven decision-making within the broader framework of Machine Learning. The significance of feature engineering, model selection, and interpretability within this symbiotic relationship is a focal point of our investigation.

2. Motivations and Literature Survey

The importance of the research is evident from the emergence of the fuzzy state of many phenomena that have a statistical relationship with each other and have inaccurate data, which calls for formulating a fuzzy regression model that represents this relationship and estimating the coefficients of that model in order to describe the fuzzy relationship efficiently. This study proposes an integrated approach that combines between the strengths of fuzzy multiple linear regression (FMLR) and machine learning techniques to enhance customer behavior prediction.

In recent years, many types of fuzzy regression models have been created for regression analysis recovery. The fuzzy linear regression model was first introduced by (Tanaka et al,1982), Their model handled after that by many authors, like (Lee,1988). These models can be roughly classified into three groups, linear programming (LP) methods (Nasrabadi M., Nasrabadi E., 2004), multi-objective (MO) techniques, and least squares (LS) methods, (Chen L. and Hsueh C. 2009). Most previous works show that both linear regression and machine learning can be effectively used for predicting and forecasting in different environments,(K.Matsumura et al 2015),(Dehghan,2015) studied the familiar methods of least square deviation and absolute deviation and used a regression model with fuzzy coefficients. In a study presented by (Helena,2016) investigated whether consumption values could predict green purchasing behavior. The examination is based on the theory of consumption values and uses fuzzy qualitative comparative analysis,(Rahman,2017) proposed a new technique to classify user behavior using fuzzy rule based system, (Henrique et al.2017) Study the adequacy of nonlinear models with simple design to predict energy consumption in a real-world smart building environment. Gopalakrishnan worked on evaluating a department store's sales and estimating future sales in order to maximize its revenues and make its brands much better and more competitive by generating customer loyalty. The technology used to forecast revenue is a deep learning linear regression algorithm (T. Gopalakrishnan,et al.,2018), (Saifil,2019)experimentally compared various traditional logistic regression classification algorithms with artificial neural network to predict customer volatility, (Hye et al. 2019) presented a new method to estimate a fuzzy regression model when the center and diffusion have a different pattern,(Jing,2019) used machine learning techniques such as decision tree, cluster analysis, and Naive Bayes algorithm to analyze customer characteristics and attributes with historical purchase records, (Rizwanullah,2020) developed a customer behavior analysis model based on the adaptive fuzzy logistic regression model, Ritesh analyzed different computational algorithms that can be used in machine learning methodologies to find out the best models. It also suggests different ways to improve the efficiency of these algorithms (Ritesh Choudhary,et al,2021). (Güzelel,2021) developed several surrogate models using multiple linear regression and machine learning algorithms to determine the output states of silica gel drying wheels to achieve balanced flow. (Doan et al.2021) developed a Mamdani based fuzzy inference model to explore customer behavior while purchasing a product,(Tran,2021)studied machine learning models and data pre-processing techniques to obtain the highest accuracy solutions in predicting customer decision. (Zhengyi,2023) Machine learning has also been applied to predicting consumer behavior,(Chen,2022) using machine learning algorithms to develop processing strategies is better than multiple linear regression in prediction.

(Gyanendra,2022) presented a comparative study of different machine learning techniques applied to the problem of predicting customer purchasing behavior, The study conducted by (Kusumadewi et al,2023) aims to build a model to solve the multiple linear regression problem using Sugeno's Fuzzy Inference System (FIS) approach, the main contribution of this study is to provide an alternative model for performing linear regression. (Zifeng,2024) created a consumption behavior prediction model based on context-aware data and support vector machine classification algorithm, (Forgács,2024) introduced a new methodology, a Sugeno-type predictive fuzzy model, which enables companies to make more accurate predictions regarding the requirements and behaviors of distinct consumer groups. (Sampathraja,2024) studied fuzzy logic in machine learning based maximum power point tracking for cost-optimized grid-connected hybrid renewable energy systems.

3. Fuzzy Sets and α – cut

In our real world, we face many sets in which elements are classified according to a description or criteria that are not clearly defined in determining and knowing the extent or degree of belonging to the element, whether the element belongs or does not belong to that classification or set, this lack of clarity that is manifested in these sets, and the phenomena that have this characteristic are included in the study and are concentrated in the fuzzy set theory based on the lack of clarity and emphasis in the elements and data (Zadeh, 1965; Ibrahim , et al., 2023).

Let $\tilde{A}$ is a subset of X which represents a universal set, and defined by membership function $\Psi_{\tilde{A}}$: X→ [0,1]. One of the concepts related to the fuzzy set is the α – cut set, it is a set containing elements x with a degree of membership not less than (α) to express the degree of belonging to the important elements, which is called the level of (α), The important membership is between two values, left and right$[x_l, x_r]$, Otherwise, it is of little importance and is often left to be outside the scope of α – cut set.

A set of cut with a degree of membership greater than α is called "strong α - cut set ", denoted by $\tilde{A}_{\alpha^+}$, The largest degree of membership is called the height of the fuzzy set, while the core is the set of elements that have a degree of membership equal to one (Kreinovich, 2021).

4. Linear Regression

Linear regression is defined as a mathematical relationship that links the variables, and this relationship is expressed in the form of a linear equation that includes the dependent variable and one or more independent variables, equation (1) explains this relationship.

$$y_i = \beta_0 + \beta_1 x_{1i} + +\beta_2 x_{2i} + \cdots + +\beta_f x_{fi} + \varepsilon_i \quad (1)$$

In matrix form, linear regression is expressed as follows:

$$\mathrm{Y} = \mathrm{XB} + \varepsilon$$

Where :

Y: vector $(n \times 1)$ of observations of dependent variable,

X: vector $(n \times p)$ of observations of independent variables,

B: vector $(p \times 1)$ Unknown parameters in the model, to be estimated,

ε: vector $(n \times 1)$ of random errors.

The above model is known as the general linear model, and for the purpose of estimating its parameters, more than one method is used (Khonglumtan, 2023). One of the most widely used methods is the method of least squares, which depends on the criterion for minimizing the sum of squares of error.

5. Fuzzy Linear Regression

In traditional regression models, there is uncertainty resulting from randomness, so the probability theory cannot be (*fuzziness*), but in the case of uncertainty resulting from fuzziness, it is used, but fuzzy set theory is used. Uncertainty results in fuzzy regression if the relationship between the independent variables and the dependent variable is fuzzy, or if the data is fuzzy. These two types lead to the following types of fuzzy regression.

1- In case the data is crisp, as in the traditional regression, and the parameters are fuzzy, the independent variables have crisp data, the parameters of the model are fuzzy, and the response variable has fuzzy data.
2- In case the parameters are crisp and the data is fuzzy, as the explanatory variables and the response variable have fuzzy data, and the parameters of the model are crisp as in the traditional regression.
3- If both the data and the parameters are fuzzy, then the explanatory variables and the response variable have fuzzy data, and the parameters of the model are fuzzy as well.

6. Fuzzy Least Squares Estimators

The fuzzy least squares method used to estimate the fuzzy regression model with fuzzy parameters and fuzzy independent variables is summarized in the following steps (Chung, 2003):
Step 1: Select different values of $\alpha -$cut, each of the values chosen cuts all the fuzzy triangular numbers for all model variables, as it cuts the triangular membership to the fuzzy number, the purpose to determine α–cut set by two values$[r_\alpha^l, r_\alpha^u]$, Which represent the new minimum and maximum values after cutting, These terms can be found by applying the two relationships shown in the formula:
$r_\alpha^l = (1-\alpha) * a + \alpha * b, r_\alpha^u = (1-\alpha) * c + \alpha * b.$ (2)
Where (a, b, c) represents the principal bounds of the triangular fuzzy number, using the same formulas for the fuzzy response variable as:
$y_\alpha^l = (1-\alpha) * y_i^a + \alpha * y_i^b, y_\alpha^u = (1-\alpha) * y_i^c + \alpha * y_i^b.$ (3)
After this cut, new values for the variables are generated, represented by the lower and upper limits, based on these values, and using the following least squares formula the minimum and maximum parameters are estimated $\beta_{j(\alpha)}^{lo}, \beta_{j(\alpha)}^{up}$:

$$\hat{\tilde{\beta}}_{j(\alpha)}^{lo} = [X^T X]^{-1} X^T \otimes Y_\alpha^l, \hat{\tilde{\beta}}_{j(\alpha)}^{up} = [X^T X]^{-1} X^T \otimes Y_\alpha^u \quad (4)$$

Step 2: A interval is chosen in the form $\alpha_\zeta \leq \alpha \leq 1$, where α_ζ represents the value of a selected cut-off level within $\alpha \in [0,1]$, then the interval $\alpha_\zeta \leq \alpha \leq 1$ is divided into several small parts, so that the difference between each two consecutive parts equal to ϵ.
Step 3: Estimate the fuzzy parameter at the cutting level $\alpha_\zeta$as interval :

$$\hat{\tilde{\beta}}_{j(\alpha\zeta)} = \left[\beta_{j(\alpha_\zeta)}^{lo}, \beta_{j(\alpha_\zeta)}^{up}\right] \quad (5)$$

From the above interval, several values are chosen, let it be (p) for each parameter.
Step 4: Depending on the triangular membership function and applying the following linear programming model:

$$\begin{aligned} &\max \alpha \\ &subject\ to \\ &\beta_{j(\alpha)}^{lo} \leq p_j \leq \beta_{j(\alpha)}^{up} \quad (6) \end{aligned}$$

$\alpha_\zeta \le \alpha \le 1$

The value of membership α is generated for each value p_j that have been selected and compensated in the linear programming model.

Step 5: Choose one of the values p_j based on the membership values that resulting from the solution of linear programming model, mostly choosing values with high membership between [0.90−1].

Finally, the fuzzy linear regression model is written as:

$$\tilde{\tilde{Y}}_{i(\alpha_\zeta)} = \tilde{\tilde{\beta}}_{0(\alpha_\zeta)} \oplus \tilde{\tilde{\beta}}_{1(\alpha_\zeta)} \otimes X_{i1(\alpha_\zeta)} \oplus \tilde{\tilde{\beta}}_{2(\alpha_\zeta)} X_{i2(\alpha_\zeta)} \oplus \dots \oplus \tilde{\tilde{\beta}}_{t(\alpha_\zeta)} \otimes X_{it(\alpha_\zeta)} \quad (7)$$

7. Descriptive of Data

This paper consists data from site "kaggle.com/datasets" for a customer contains independent variable y as Spending Score and two independent variables X_1 as Age, X_2 as Annual Income and with 200 observations. Ensure the dataset includes the target variable and the features to use for prediction, where y is fuzzy triangular numbers as shown in Appendix (A).

8. Estimation of Parameters of fuzzy regression models

The estimation of central data is based on the results of the linear programming model (6) as shown in Appendix (B), which depends on the lower and upper least squares estimators. The levels within the interval [0.90−1] are selected with their corresponding estimated parameters in the linear programming model. The estimated of central parameters of the fuzzy regression model shown in the following table:

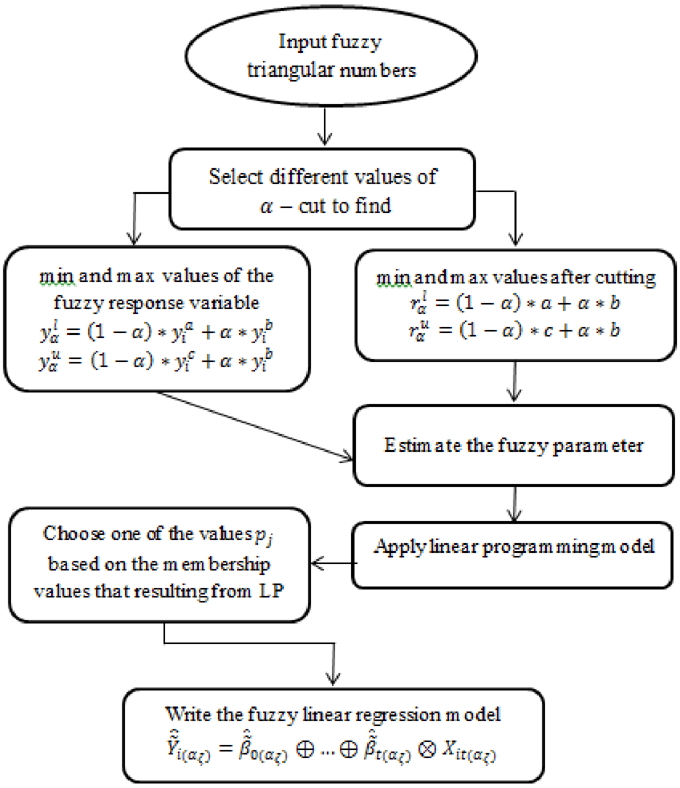

Fig.1: Flow chart to illustrate Fuzzy Least Squares Estimators

α_ζ	$\tilde{\tilde{\beta}}^m_{0(\alpha_\zeta)}$	$\tilde{\tilde{\beta}}^m_{1(\alpha_\zeta)}$	$\tilde{\tilde{\beta}}^m_{2(\alpha_\zeta)}$
0.90	73.552	-0.60409	0.006012
0.91	73.329	-0.60420	0.006010
0.92	73.218	-0.60430	0.006080
0.93	73.348	-0.60409	0.006075

0.94	73.503	-0.60405	0.006170
0.95	73.366	-0.60410	0.006250
0.96	73.406	-0.60500	0.006340
0.97	73.371	-0.60500	0.006180
0.98	73.335	-0.60500	0.006470
0.99	73.342	-0.60500	0.00614

The results of estimating the parameters of the fuzzy regression model at the lower and upper limits and for different $\alpha_\zeta -$ cut levels are shown in the following table.

α_ζ	$\hat{\tilde{\beta}}^l_{0(\alpha_\zeta)}, \hat{\tilde{\beta}}^u_{0(\alpha_\zeta)}$	$\hat{\tilde{\beta}}^l_{1(\alpha_\zeta)}, \hat{\tilde{\beta}}^u_{1(\alpha_\zeta)}$	$\hat{\tilde{\beta}}^l_{2(\alpha_\zeta)}, \hat{\tilde{\beta}}^u_{2(\alpha_\zeta)}$
0.90	(73.32,73.904)	(-0.604,-0.6042)	(0.006,0.0070)
0.91	(73.192, 73.501)	(-0.604,-0.605)	(0.006,0.0070)
0.92	(73.209,73.484)	(-0.604,-0.605)	(0.006,0.0070)
0.93	(73.227,73.467)	(-0.604,-0.605)	(0.006,0.0069)
0.94	(73.244,74.917)	(-0.604,-0.617)	(0.006,0.0069)
0.95	(73.261,73.433)	(-0.604,-0.605)	(0.006,0.0068)
0.96	(73.279,73.416)	(-0.605,-0.605)	(0.006,0.0068)
0.97	(73.296,73.399)	(-0.605,-0.605)	(0.006,0.0066)
0.98	(73.313,73.382)	(-0.605,-0.605)	(0.006,0.0065)
0.99	(73.331,73.365)	(-0.605,-0.605)	(0.006,0.0065)

The fuzzy set that consists of multiple values (triplets) is converted into an output with a single value. This process is known as *defuzzification*. One of these methods is "*The graded mean integration representation"***,** where the crisp number is calculated by (Espinosa, et al., 2005):

$$x_{cr} = \frac{a+2b+c}{4} \quad (7)$$

α_ζ	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
$x^{\beta_0}_{cr}$	73.582	73.338	73.282	73.347	73.792	73.357	73.377	73.359	73.341	73.34
$x^{\beta_1}_{cr}$	-0.6040	-0.6044	-0.6044	-.60435	-0.6073	-0.6043	-0.605	-0.605	-0.605	-0.605
$x^{\beta_2}_{cr}$	0.00626	0.00625	0.00629	0.00626	0.00631	0.00633	0.00637	0.00624	0.00636	0.0062

When taking the alpha cut at level ($\alpha_\zeta = 0.95$), the regression equation is written as follows:

$$\hat{\tilde{Y}}_{i(0.95)} = 73.357 \oplus -0.6043 \otimes X_{i1(0.95))} \oplus 0.00633 X_{i2(0.95)} \oplus \epsilon_i \quad (8)$$

The parameters of the multiple linear regression equation in equation (8), explain the effect of the two independent variables, age and annual income, on the dependent variable expense, where the value of $\hat{\tilde{\beta}}_1$=-0.6043, It means that when the consumer's age increases by one unit, the spending decreases by this amount (i.e. there is a reverse effect), and the value of $\hat{\tilde{\beta}}_2$=0.00633, It means that when the consumer's income increases by one unit, the spending increases by this amount (i.e. there is a positive effect).

9. Multiple linear regression in machine learning

Multiple linear regression is a statistical technique that can be used to predict the value of a dependent variable based on the values of two or more independent variables. It is a supervised learning algorithm, which means that it learns from a dataset of labeled examples, where the dependent variable is known.

Once the model has been trained, it can be used to predict the value of the dependent variable for new data points. This can be used for a variety of tasks, such as predicting house prices, customer churn, or medical risk (El-Naggar, 2023).

10.Machine Learning Approach

While MLR is traditionally viewed as a statistical technique, it can also be implemented within the machine learning framework. Here's how:

10.1. Data Preparation:

- Collect and pre-process data, ensuring numerical features and addressing missing values and outliers.
- Feature scaling may be necessary for variables with vastly different scales to avoid biasing the coefficients.

10.2. Training Mode:

During training, the model ingests the training data. This data consists of features (inputs) and targets (outputs). The model's internal parameters are adjusted based on the observed patterns in the training data. Here's a breakdown of the theory:

Loss Function: The model makes predictions on the training data. These predictions are compared to the actual targets using a loss function. This function quantifies the discrepancy between prediction and reality. Examples include squared error for regression problems or cross-entropy for classification problems.

Optimization Algorithm: The model aims to minimize the overall loss across all training examples. This is achieved by an optimization algorithm that iteratively adjusts the model's internal parameters. Common algorithms include gradient descent and its variants. These algorithms take the calculated loss and use it to update the parameters in a way that reduces the loss in subsequent iterations.

Essentially, the training process allows the model to learn a mapping function between features and targets.

10.3. Testing Mode:

Once trained, the model's performance is evaluated on unseen data – the testing set. This helps assess how well the model generalizes to real-world scenarios beyond the training data it was exposed to during training. Here's the theory behind testing:

Generalization: Ideally, the model's performance on the testing data should be similar to its performance on the training data. A significant drop in performance indicates over fitting, where the model has memorized the training data but fails to capture underlying patterns. Techniques like regularization can help mitigate over fitting.

Error Analysis: Analyzing errors on the testing data can reveal specific weaknesses in the model. This can guide further improvements in data preprocessing, feature engineering, or even exploration of different model architectures.

10.4. Model Evaluation:

Evaluate the model's performance using the testing dataset. Common evaluation metrics for regression problems include Mean Squared Error (MSE).

11. Results and discussions

Interpreting the results of using machine learning for linear regression to predict customer behavior is an important step in gaining meaningful insights from data (Fig.2).

- Predicting the Test set results

```
y_pred = regressor.predict(X_test)
print("prediction = ",y_pred)
```

```
prediction =  [43.92715471 48.34781432 41.98513652 45.00424686 54.76235252 45.11461501
 58.92755576 44.62364426 41.02698268 46.58087994 59.72348104 53.20950711
 49.5957052  45.09511243 40.71345798 58.40851226 60.5279765  45.26162607
 54.5611188  39.72294631 53.53396422 54.52876098 57.4137155  53.51446164
 34.46543342 49.27124808 50.68136752 53.01492072 48.77599225 45.03231961
 52.39407924 45.92768063 47.07613578 59.31909078 46.8511144  49.94823506
 49.71892859 54.47025324 46.16127217 36.04206649]
```

Fig.2: predict customer behavior using Python Code.

Machine learning algorithms can be used to determine the models and tendency enclosed by data enable businesses to best understand how their customers interact with products or services. By examine the output of a fuzzy linear regression model, we can obtain great insight into customer behaviors and desire that would remain fluctuates. We use the mean square error (MSE), which calculates the average squared difference between the predicted values and the actual values. See Figure (3)

```
# Calculation of Mean Squared Error (MSE)
mean_squared_error(y_test,y_pred)
```

```
623.7455096905665
```

Fig. 3: Calculation of Mean Square Error using Python Code.

When this measure have been realized it's possible to estimate whether our predictive models unambiguously gain the different in customer behavior.

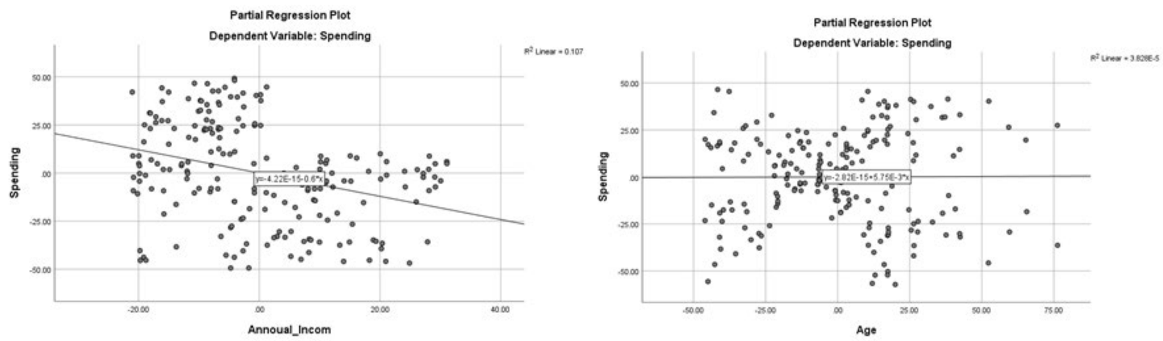

Fig. 4: Partial Regression plots by dependent var. spending and independent variables annual income and age.

12.Conclusion

This study has presented a novel approach to customer behavior prediction by integrating fuzzy multiple linear regression (FMLR) with machine learning techniques. The proposed method addresses the challenges posed by imprecise or fuzzy customer data, which traditional regression methods often struggle with. By combining the ability of FMLR to handle fuzziness with the powerful model training and optimization capabilities of machine learning, the developed framework offers a robust and interpretable solution for predicting customer spending patterns.

The application of this approach to a real customer dataset demonstrated its potential in capturing the complex relationships between customer characteristics (age and income) and their spending behavior. The estimated fuzzy regression model, along with the optimal α-cut level selection, provides businesses with actionable insights into customer preferences, purchasing tendencies, and opportunities for targeted marketing and cross-selling strategies.

While the results are promising, further research is needed to explore the generalizability of this approach across different industry sectors and customer behavioral domains. Additionally, the integration of other machine learning techniques, such as neural networks or ensemble methods, could be investigated to potentially enhance the predictive performance further.

In conclusion, this study contributes to the field by introducing a synergistic combination of FMLR and machine learning, opening new avenues for businesses to gain a deeper understanding of their customers and make more informed, data-driven decisions for improved customer relationships and market competitiveness.

References

Anett Forgács, et al.(2024). Towards the Investigation of Online Shopping Behaviors Using a Fuzzy Inference System Decision Making: Applications in Management and Engineering, Volume 7, Issue 2 , 337-354. doi.org/10.31181/dmame7220241059

Atta-ur-Rahman1, Saleh A. Alrashed2, Ajith Abraham,(2017). User Behaviour Classification and Prediction Using Fuzzy Rule Based System and Linear Regression, Journal of Information Assurance and Security. Vol. 11, 086-093

Bahzad T.Salim, Mohammed K. Issa.2024, The Impact of Digital Marketing Management on Customers Buying Behavior: Case Study of Soran City, Entrepreneurship Journal for Finance and Business, vol. 05, No. 01, 91-112.doi.org/10.56967/ejfb2024379

Doan Van Thang et al.(2021). A fuzzy-based expert system to analyses purchase behavior under uncertain environment, Institute of Computer Applications and Management, Volume 13, pages 997–1004

Espinosa J. , Vandewalle J. & Wertz V. (2005) , Fuzzy Logic, Identification and predictive control , Springer sciences, Business Media , LLC.doi:10.1007/b138626

Gyanendra Chaubey et al.(2022), Customer purchasing behavior prediction using machine learning classification techniques, Journal of Ambient Intelligence and Humanized Computing doi.org/10.1007/s12652-022-03837-6.

Helena Martins Gonçalves, Tiago Ferreira Lourenço , Graça Miranda Silva,(2016). Green buying behavior and the theory of consumption values: A fuzzy-set approach, Journal of Business Research Vol. 69, Issue 4, 1484-1491

Henrique Pombeiro et al. (2017) "Comparative assessment of low-complexity models to predict electricity consumption in an institutional building: Linear regression vs. fuzzy modeling vs. neural networks". eng. In: Energy & Buildings 146, pp. 141–151.

Hye-Young Jung, Woo-Joo Lee and Seung Hoe Choi,(2019). Fuzzy regression model using fuzzy partition, Journal of Physics: Conf. Series 1334.

Ibrahim Ali Abdulhafidh, Bilal N. Saeed and Marwa A. Fadil, (2023), Forecasting Stock Prices with an Integrated Approach Combining ARIMA and Machine Learning Techniques ARIMAML, Journal of Computer and Communications, 11, 58-70. doi:10.4236/jcc.2023.118005

Ibrahim Ali Abdulhafidh, (2024), Data Science using Python Language with Applications (book),Dar Altheaker,1st edition.

K. Matsumura, Carlos F. Gaitan, K. Sugimotoand William Hsieh(2014) “Maize yield forecasting by linear regres-sion and artificial neural networks in Jilin, China”. In: The Journalof Agricultural Science. doi: 10.1017/S0021859614000392.

Jafar Raed, and Adel Awad, Iyad Hatem, (2023), Multiple Linear Regression and Machine Learning for Predicting the Drinking Water Quality Index in Al-Seine Lake, Smart Cities, 6, 2807–2827.doi.org/10.3390/smartcities6050126

Jing LI, Shuxiao PAN, Lei HUANG, Xin ZHU,(2019). A Machine Learning Based Method for Customer Behavior Prediction, Technical Gazette vol.26, No.6, 1670-1676.

L.A. Zadeh, (1965). Fuzzy sets, Information and Control 8, 338-353. doi.org/10.1016/S0019-9958(65)90241

Liang-Hsuan Chen and Chan Ching Hsueh,(2009) Fuzzy Regression Models Using the Least-Squares Method Based on the Concept of Distance, IEEE Transactions On Fuzzy Systems, 17, 6.10.

Muhammad Faisal Ibrahim, Taufik Kurrahman, Dana Marsetiya Utama, (2023). Open-Source ERP Systems Selection: An Integrated Method based on Fuzzy AHP –TOPSIS. Journal of Logistics, Informatics and Service Science, Vol.10 No.4, 234-249.doi:10.33168/ JLISS.2023.0416

Mohammad hossein Dehghan and et al.(2015), Study of linear regression based on least squares and fuzzy least absolutes deviations and its application in geography. Conference: 2015 4th Iranian Joint Congress on Fuzzy and Intelligent Systems. doi:10.1109/CFIS.2015.7391667

Mohd Rizwanullah, Salah Abunar, Sayeeduzzafar Qazi(2020). Customer satisfaction and behavior at retail outlet: an adaptive fuzzy regression model with lingo based analysis, Marketing and Management of Innovations 2, 275-285. doi.org/10.21272/mmi.2020.2-20.

N. Sampathraja and L. Ashok Kumar,(2024). Fuzzy logic inherited machine learning based maximum power point tracker for cost-optimized grid connected hybrid renewable systems, Iranian Journal of Fuzzy Systems, Vol. 21, No. 1, 103-128. doi.org/10.22111/IJFS.2023.44709.7874.

Nasrabadi M.M., Nasrabadi E. (2004). A mathematical-programming approach to fuzzy linear regression analysis, applied mathematics and computation, 155,3-881, doi: 10.1016/j.amc .2003.07.031

Ritesh Choudhary, T Gopalakrishnan, D Ruby, A Gayathri, Vishnu Srinivasa Murthy, Rishabh Shekhar, (2021) An Efficient Model for predicting liver disease using machine learning, Data Analytics in Bioinformatics: A Machine Learning Perspective, John Wiley & Sons, Inc.

Saifil Momin, Bohra T, Raut P (2019) Prediction of customer churn using machine learning, In EAI international conference on big data innovation for sustainable cognitive computing, Springer International Publishing, pp 203–212.

Sri Kusumadewi, Linda Rosita, Elyza Gustri Wahyuni,(2023). Fuzzy linear regression based on a hybrid of fuzzy C-means and the fuzzy inference system for predicting serum iron levels in patients with chronic kidney disease. Expert Systems with Applications Volume 227, doi.org/10.1016/j.eswa.2023.120314Get

Tara Khonglumtan, Parinya Srisattayaku ,(2023).Applying Ordinal Logistic Regression to Analyze the Service Confidence for an Air Conditioner Service Center. Journal of Logistics, Informatics and Service Science, Vol. 10 No. 1, 72-93 doi:10.33168/JLISS.2023.0205

T. Gopalakrishnan, Ritesh Choudhary, Sarada Prasad,(2018) Prediction of sales value in online shopping using linear regression, 2018 4th International Conference on Computing Communication and Automation (ICCCA).

Tiryaki Ozde, and Sakar C. Okan, (2018).Nonlinear Feature Extraction using Multilayer Perceptron based Alternating Regression for Classification and Multiple-output Regression Problems.7th International Conference on Data Science, Technology and Applications.107-117. doi.org: 10.5220/ 00068 489010 70117.

Tanaka H., and Lee H. (1988). Interval regression analysis by quadratic programming approach . IEEE Trans. Systems Man Cybrnet., 6(4), 473-481.

Tanaka H., Uegima S., and Asai K. (1982). Linear regression analysis with fuzzy model , IEEE Systems Man Cybrnet., 12(6), 903-907.

Tran Duc Quynh and Hoang Thi Thuy Dung,(2021). Prediction of Customer Behavior using Machine Learning: A Case Study, Proceedings of the 2nd International Conference on Human-centered Artificial Intelligence 28-October-2021.

Wang Feng, Aviles Jeoy (2023), Enhancing Operational Efficiency: Integrating Machine Learning Predictive Capabilities in Business Intelligence for Informed Decision-Making. doi.org 10.54097/fbem.v9i1.8694

Wu, Hsien Chung, (2003). Linear regression analysis for fuzzy input and output data using the extension principle, Computers and Mathematics with Applications vol.45 No.12, 1849-1859,doi.org/ 10.1016/ S0898-1221(03)90006.

Vladik Kreinovich, (2021) An Introduction to Computing with Fuzzy Sets, Analysis, Design, and Applications, Journal of Intelligent & Fuzzy Systems, Springer Nature Switzerland, 40, 1717–1719.doi:10.3233/JIFS-189473

X. Chen , H. Zheng, H. Wang, T. Yan,(2022). Machine learning algorithms perform better than multiple linear regression in predicting manure nitrogen output from lactating dairy cows, Animal - science proceedings Volume 13, Issue 1, doi.org/10.1016/j.anscip.2022.03.069.

Yunus Emre Güzelel, Umutcan Olmuş, Kamil Neyfel Çerçi,(2021). Comprehensive modeling of rotary desiccant wheel with different multiple regression and machine learning methods for balanced flow, Applied Thermal Engineering Vol. 199,No.25, doi.org/10.1016/j.applthermaleng.

Zhengyi Hu, (2023). Consumer Behavior Prediction Based on Machine Learning Scenarios. AHIS 5, pp. 410–419 doi.org/10.2991/978-94-6463-030-5_ 43

Zifeng Zhang,(2024). Consumer behavior prediction and marketing strategy optimization based on big data Analysis, Applied Mathematics and Nonlinear Sciences, 9(1) (2024) 1-14

https://www.kaggle.com/datasets/dhiyamahdiyah/mall-customers1.

Appendix A

Customer ID	Gender	Age	Annual Income (k$)	Spending Score by fuzzy triangular number (1-100)			Customer ID	Gender	Age	Annual Income (k$)	Spending Score by fuzzy triangular number (1-100)		
				y_i^a	y_i^b	y_i^c					y_i^a	y_i^b	y_i^c
1	Male	19	15	38	39	42	34	Male	18	33	90	92	95
2	Male	21	15	79	81	84	35	Female	49	33	12	14	15
3	Female	20	16	6	6	9	36	Female	21	33	79	81	81
4	Female	23	16	75	77	78	37	Female	42	34	15	17	20
5	Female	31	17	39	40	40	38	Female	30	34	70	73	74
6	Female	22	17	75	76	80	39	Female	36	37	25	26	29
7	Female	35	18	5	6	8	40	Female	20	37	73	75	76
8	Female	23	18	90	94	96	41	Female	65	38	34	35	39
9	Male	64	19	3	3	5	42	Male	24	38	90	92	92
10	Female	30	19	70	72	73	43	Male	48	39	33	36	37
11	Male	67	19	12	14	18	44	Female	31	39	60	61	65
12	Female	35	19	98	99	100	45	Female	49	39	27	28	28
13	Female	58	20	15	15	17	46	Female	24	39	64	65	69
14	Female	24	20	76	77	80	47	Female	50	40	51	55	58
15	Male	37	20	10	13	14	48	Female	27	40	46	47	49
16	Male	22	20	77	79	84	49	Female	29	40	42	42	44
17	Female	35	21	33	35	36	50	Female	31	40	40	42	45
18	Male	20	21	65	66	69	51	Female	49	42	50	52	55
19	Male	52	23	27	29	33	52	Male	33	42	57	60	61
20	Female	35	23	95	98	99	53	Female	31	43	56	54	58
21	Male	35	24	34	35	37	54	Male	59	43	59	60	63
22	Male	25	24	70	73	74	55	Female	50	43	45	45	48
23	Female	46	25	5	5	7	56	Male	47	43	38	41	41
24	Male	31	25	72	73	75	57	Female	51	44	49	50	52
25	Female	54	28	14	14	16	58	Male	69	44	42	46	47
26	Male	29	28	80	82	83	59	Female	27	46	50	51	54
27	Female	45	28	30	32	33	60	Male	53	46	44	46	46
28	Male	35	28	60	61	63	61	Male	70	46	55	56	59
29	Female	40	29	31	31	33	62	Male	19	46	52	55	59
30	Female	23	29	84	87	88	63	Female	67	47	50	52	55
31	Male	60	30	3	4	6	64	Female	54	47	59	59	62
32	Female	21	30	72	73	75	65	Male	63	48	50	51	54
33	Male	53	33	4	4	5	66	Male	18	48	55	59	60
Customer ID	Gender	Age	Annual Income (k$)	Spending Score by fuzzy triangular number (1-100)			Customer ID	Gender	Age	Annual Income (k$)	Spending Score by fuzzy triangular number (1-100)		
				y_i^a	y_i^b	y_i^c					y_i^a	y_i^b	y_i^c
67	Female	43	48	49	50	55	106	Female	21	62	39	42	42
68	Female	68	48	46	48	53	107	Female	66	63	48	50	54
69	Male	19	48	58	59	59	108	Male	54	63	44	46	50
70	Female	32	48	45	47	50	109	Male	68	63	43	43	45

71	Male	70	49	54	55	59	110	Male	66	63	47	48	51
72	Female	47	49	41	42	45	111	Male	65	63	47	52	52
73	Female	60	50	46	49	50	112	Female	19	63	50	54	55
74	Female	60	50	56	56	60	113	Female	38	64	41	42	46
75	Male	59	54	47	47	50	114	Male	19	64	45	46	48
76	Male	26	54	53	54	57	115	Female	18	65	46	48	49
77	Female	45	54	53	53	55	116	Female	19	65	50	50	53
78	Male	40	54	45	48	50	117	Female	63	65	40	43	44
79	Female	23	54	50	52	55	118	Female	49	65	57	59	63
80	Female	49	54	41	42	45	119	Female	51	67	40	43	43
81	Male	57	54	51	51	53	120	Female	50	67	55	57	60
82	Male	38	54	50	55	56	121	Male	27	67	56	56	60
83	Male	67	54	40	41	41	122	Female	38	67	35	40	42
84	Female	46	54	43	44	47	123	Female	40	69	57	58	59
85	Female	21	54	56	57	60	124	Male	39	69	90	91	94
86	Male	48	54	46	46	50	125	Female	23	70	29	29	32
87	Female	55	57	53	58	60	126	Female	31	70	75	77	77
88	Female	22	57	54	55	58	127	Male	43	71	36	35	38
89	Female	34	58	59	60	64	128	Male	40	71	94	95	99
90	Female	50	58	45	46	49	129	Male	59	71	10	11	13
91	Female	68	59	55	55	56	130	Male	38	71	74	75	77
92	Male	18	59	40	41	44	131	Male	47	71	9	9	11
93	Male	48	60	49	49	50	132	Male	39	71	74	75	78
94	Female	40	60	39	40	43	133	Female	25	72	33	34	38
95	Female	32	60	42	42	44	134	Female	31	72	70	71	74
96	Male	24	60	51	52	55	135	Male	20	73	5	5	8
97	Female	47	60	47	47	49	136	Female	29	73	86	88	89
98	Female	27	60	50	50	54	137	Female	44	73	7	7	9
99	Male	48	61	39	42	44	138	Male	32	73	73	73	78
100	Male	20	61	45	49	50	139	Male	19	74	9	10	14
101	Female	23	62	38	41	42	140	Female	35	74	70	72	75
102	Female	49	62	47	48	52	141	Female	57	75	5	5	9
103	Male	67	62	57	59	62	142	Male	32	75	90	93	97
104	Male	26	62	55	55	57	143	Female	28	76	39	40	44
105	Male	49	62	56	56	59	144	Female	32	76	86	87	87
Customer ID	**Gender**	**Age**	**Annual Income (k$)**	**Spending Score by fuzzy triangular number (1-100)**			**Customer ID**	**Gender**	**Age**	**Annual Income (k$)**	**Spending Score by fuzzy triangular number (1-100)**		
				y_i^a	y_i^b	y_i^c					y_i^a	y_i^b	y_i^c
145	Male	25	77	14	12	15	173	Male	36	87	10	10	14
146	Male	28	77	96	97	101	174	Male	36	87	91	92	94
147	Male	48	77	35	36	38	175	Female	52	88	12	13	16
148	Female	32	77	74	74	77	176	Female	30	88	84	86	90
149	Female	34	78	21	22	25	177	Male	58	88	14	15	17
150	Male	34	78	89	90	93	178	Male	27	88	68	69	72
151	Male	43	78	16	17	20	179	Male	59	93	13	14	17

152	Male	39	78	87	88	90	180	Male	35	93	86	90	92
153	Female	44	78	20	20	22	181	Female	37	97	30	32	33
154	Female	38	78	73	76	77	182	Female	32	97	86	86	89
155	Female	47	78	15	16	18	183	Male	46	98	14	15	19
156	Female	27	78	87	89	93	184	Female	29	98	87	88	90
157	Male	37	78	1	1	2	185	Female	41	99	39	39	40
158	Female	30	78	76	78	82	186	Male	30	99	94	97	100
159	Male	34	78	1	1	1	187	Female	54	101	23	24	28
160	Female	30	78	70	73	73	188	Male	28	101	68	68	72
161	Female	56	79	35	35	37	189	Female	41	103	15	17	20
162	Female	29	79	83	83	86	190	Female	36	103	83	85	89
163	Male	19	81	5	5	7	191	Female	34	103	22	23	28
164	Female	31	81	90	93	97	192	Female	32	103	66	69	70
165	Male	50	85	26	26	30	193	Male	33	113	8	8	9
166	Female	36	85	72	75	77	194	Female	38	113	90	91	94
167	Male	42	86	20	20	24	195	Female	47	120	16	16	20
168	Female	33	86	91	95	95	196	Female	35	120	75	79	79
169	Female	36	87	26	27	29	197	Female	45	126	24	28	30
170	Male	32	87	61	63	66	198	Male	32	126	73	74	79
171	Male	40	87	13	13	18	199	Male	32	137	17	18	22
172	Male	28	87	73	75	77	200	Male	30	137	80	83	88

Appendix B

$y^l_{i(.05)}$	$y^u_{i(.05)}$	$y^l_{i(.15)}$	$y^u_{i(.15)}$	$y^l_{i(.25)}$	$y^u_{i(.25)}$	$y^l_{i(.35)}$	$y^u_{i(.35)}$	$y^l_{i(.45)}$	$y^u_{i(.45)}$	$y^l_{i(.55)}$	$y^u_{i(.55)}$	$y^l_{i(.65)}$	$y^u_{i(.65)}$	$y^l_{i(.75)}$	$y^u_{i(.75)}$	$y^l_{i(.85)}$	$y^u_{i(.85)}$
38.05	41.85	38.15	39.45	38.25	41.25	38.35	40.95	38.45	40.65	38.55	40.35	38.65	40.05	38.75	39.75	38.85	39.45
79.1	83.85	79.3	81.45	79.5	83.25	79.7	82.95	79.9	82.65	80.1	82.35	80.3	82.05	80.5	81.75	80.7	81.45
6	8.85	6	6.45	6	8.25	6	7.95	6	7.65	6	7.35	6	7.05	6	6.75	6	6.45
75.1	77.95	75.3	77.15	75.5	77.75	75.7	77.65	75.9	77.55	76.1	77.45	76.3	77.35	76.5	77.25	76.7	77.15
39.05	40	39.15	40	39.25	40	39.35	40	39.45	40	39.55	40	39.65	40	39.75	40	39.85	40
75.05	79.8	75.15	76.6	75.25	79	75.35	78.6	75.45	78.2	75.55	77.8	75.65	77.4	75.75	77	75.85	76.6
5.05	7.9	5.15	6.3	5.25	7.5	5.35	7.3	5.45	7.1	5.55	6.9	5.65	6.7	5.75	6.5	5.85	6.3
90.2	95.9	90.6	94.3	91	95.5	91.4	95.3	91.8	95.1	92.2	94.9	92.6	94.7	93	94.5	93.4	94.3
3	4.9	3	3.3	3	4.5	3	4.3	3	4.1	3	3.9	3	3.7	3	3.5	3	3.3
70.1	72.95	70.3	72.15	70.5	72.75	70.7	72.65	70.9	72.55	71.1	72.45	71.3	72.35	71.5	72.25	71.7	72.15
12.1	17.8	12.3	14.6	12.5	17	12.7	16.6	12.9	16.2	13.1	15.8	13.3	15.4	13.5	15	13.7	14.6
98.05	99.95	98.15	99.15	98.25	99.75	98.35	99.65	98.45	99.55	98.55	99.45	98.65	99.35	98.75	99.25	98.85	99.15
15	16.9	15	15.3	15	16.5	15	16.3	15	16.1	15	15.9	15	15.7	15	15.5	15	15.3
76.05	79.85	76.15	77.45	76.25	79.25	76.35	78.95	76.45	78.65	76.55	78.35	76.65	78.05	76.75	77.75	76.85	77.45
10.15	13.95	10.45	13.15	10.75	13.75	11.05	13.65	11.35	13.55	11.65	13.45	11.95	13.35	12.25	13.25	12.55	13.15
77.1	83.75	77.3	79.75	77.5	82.75	77.7	82.25	77.9	81.75	78.1	81.25	78.3	80.75	78.5	80.25	78.7	79.75
33.1	35.95	33.3	35.15	33.5	35.75	33.7	35.65	33.9	35.55	34.1	35.45	34.3	35.35	34.5	35.25	34.7	35.15
65.05	68.85	65.15	66.45	65.25	68.25	65.35	67.95	65.45	67.65	65.55	67.35	65.65	67.05	65.75	66.75	65.85	66.45
27.1	32.8	27.3	29.6	27.5	32	27.7	31.6	27.9	31.2	28.1	30.8	28.3	30.4	28.5	30	28.7	29.6
95.15	98.95	95.45	98.15	95.75	98.75	96.05	98.65	96.35	98.55	96.65	98.45	96.95	98.35	97.25	98.25	97.55	98.15
34.05	36.9	34.15	35.3	34.25	36.5	34.35	36.3	34.45	36.1	34.55	35.9	34.65	35.7	34.75	35.5	34.85	35.3
70.15	73.95	70.45	73.15	70.75	73.75	71.05	73.65	71.35	73.55	71.65	73.45	71.95	73.35	72.25	73.25	72.55	73.15
5	6.9	5	5.3	5	6.5	5	6.3	5	6.1	5	5.9	5	5.7	5	5.5	5	5.3

72.05	74.9	72.15	73.3	72.25	74.5	72.35	74.3	72.45	74.1	72.55	73.9	72.65	73.7	72.75	73.5	72.85	73.3
14	15.9	14	14.3	14	15.5	14	15.3	14	15.1	14	14.9	14	14.7	14	14.5	14	14.3
80.1	82.95	80.3	82.15	80.5	82.75	80.7	82.65	80.9	82.55	81.1	82.45	81.3	82.35	81.5	82.25	81.7	82.15
30.1	32.95	30.3	32.15	30.5	32.75	30.7	32.65	30.9	32.55	31.1	32.45	31.3	32.35	31.5	32.25	31.7	32.15
60.05	62.9	60.15	61.3	60.25	62.5	60.35	62.3	60.45	62.1	60.55	61.9	60.65	61.7	60.75	61.5	60.85	61.3
31	32.9	31	31.3	31	32.5	31	32.3	31	32.1	31	31.9	31	31.7	31	31.5	31	31.3
84.15	87.95	84.45	87.15	84.75	87.75	85.05	87.65	85.35	87.55	85.65	87.45	85.95	87.35	86.25	87.25	86.55	87.15
3.05	5.9	3.15	4.3	3.25	5.5	3.35	5.3	3.45	5.1	3.55	4.9	3.65	4.7	3.75	4.5	3.85	4.3
72.05	74.9	72.15	73.3	72.25	74.5	72.35	74.3	72.45	74.1	72.55	73.9	72.65	73.7	72.75	73.5	72.85	73.3
4	4.95	4	4.15	4	4.75	4	4.65	4	4.55	4	4.45	4	4.35	4	4.25	4	4.15
90.1	94.85	90.3	92.45	90.5	94.25	90.7	93.95	90.9	93.65	91.1	93.35	91.3	93.05	91.5	92.75	91.7	92.45
12.1	14.95	12.3	14.15	12.5	14.75	12.7	14.65	12.9	14.55	13.1	14.45	13.3	14.35	13.5	14.25	13.7	14.15
79.1	81	79.3	81	79.5	81	79.7	81	79.9	81	80.1	81	80.3	81	80.5	81	80.7	81
15.1	19.85	15.3	17.45	15.5	19.25	15.7	18.95	15.9	18.65	16.1	18.35	16.3	18.05	16.5	17.75	16.7	17.45
70.15	73.95	70.45	73.15	70.75	73.75	71.05	73.65	71.35	73.55	71.65	73.45	71.95	73.35	72.25	73.25	72.55	73.15
$y^l_{i(.05)}$	$y^u_{i(.05)}$	$y^l_{i(.15)}$	$y^u_{i(.15)}$	$y^l_{i(.25)}$	$y^u_{i(.25)}$	$y^l_{i(.35)}$	$y^u_{i(.35)}$	$y^l_{i(.45)}$	$y^u_{i(.45)}$	$y^l_{i(.55)}$	$y^u_{i(.55)}$	$y^l_{i(.65)}$	$y^u_{i(.65)}$	$y^l_{i(.75)}$	$y^u_{i(.75)}$	$y^l_{i(.85)}$	$y^u_{i(.85)}$
25.05	28.85	25.15	26.45	25.25	28.25	25.35	27.95	25.45	27.65	25.55	27.35	25.65	27.05	25.75	26.75	25.85	26.45
73.1	75.95	73.3	75.15	73.5	75.75	73.7	75.65	73.9	75.55	74.1	75.45	74.3	75.35	74.5	75.25	74.7	75.15
34.05	38.8	34.15	35.6	34.25	38	34.35	37.6	34.45	37.2	34.55	36.8	34.65	36.4	34.75	36	34.85	35.6
90.1	92	90.3	92	90.5	92	90.7	92	90.9	92	91.1	92	91.3	92	91.5	92	91.7	92
33.15	36.95	33.45	36.15	33.75	36.75	34.05	36.65	34.35	36.55	34.65	36.45	34.95	36.35	35.25	36.25	35.55	36.15
60.05	64.8	60.15	61.6	60.25	64	60.35	63.6	60.45	63.2	60.55	62.8	60.65	62.4	60.75	62	60.85	61.6
27.05	28	27.15	28	27.25	28	27.35	28	27.45	28	27.55	28	27.65	28	27.75	28	27.85	28
64.05	68.8	64.15	65.6	64.25	68	64.35	67.6	64.45	67.2	64.55	66.8	64.65	66.4	64.75	66	64.85	65.6
51.2	57.85	51.6	55.45	52	57.25	52.4	56.95	52.8	56.65	53.2	56.35	53.6	56.05	54	55.75	54.4	55.45
46.05	48.9	46.15	47.3	46.25	48.5	46.35	48.3	46.45	48.1	46.55	47.9	46.65	47.7	46.75	47.5	46.85	47.3
42	43.9	42	42.3	42	43.5	42	43.3	42	43.1	42	42.9	42	42.7	42	42.5	42	42.3
40.1	44.85	40.3	42.45	40.5	44.25	40.7	43.95	40.9	43.65	41.1	43.35	41.3	43.05	41.5	42.75	41.7	42.45
50.1	54.85	50.3	52.45	50.5	54.25	50.7	53.95	50.9	53.65	51.1	53.35	51.3	53.05	51.5	52.75	51.7	52.45
57.15	60.95	57.45	60.15	57.75	60.75	58.05	60.65	58.35	60.55	58.65	60.45	58.95	60.35	59.25	60.25	59.55	60.15
55.9	57.8	55.7	54.6	55.5	57	55.3	56.6	55.1	56.2	54.9	55.8	54.7	55.4	54.5	55	54.3	54.6
59.05	62.85	59.15	60.45	59.25	62.25	59.35	61.95	59.45	61.65	59.55	61.35	59.65	61.05	59.75	60.75	59.85	60.45
45	47.85	45	45.45	45	47.25	45	46.95	45	46.65	45	46.35	45	46.05	45	45.75	45	45.45
38.15	41	38.45	41	38.75	41	39.05	41	39.35	41	39.65	41	39.95	41	40.25	41	40.55	41
49.05	51.9	49.15	50.3	49.25	51.5	49.35	51.3	49.45	51.1	49.55	50.9	49.65	50.7	49.75	50.5	49.85	50.3
42.2	46.95	42.6	46.15	43	46.75	43.4	46.65	43.8	46.55	44.2	46.45	44.6	46.35	45	46.25	45.4	46.15
50.05	53.85	50.15	51.45	50.25	53.25	50.35	52.95	50.45	52.65	50.55	52.35	50.65	52.05	50.75	51.75	50.85	51.45
44.1	46	44.3	46	44.5	46	44.7	46	44.9	46	45.1	46	45.3	46	45.5	46	45.7	46
55.05	58.85	55.15	56.45	55.25	58.25	55.35	57.95	55.45	57.65	55.55	57.35	55.65	57.05	55.75	56.75	55.85	56.45
52.15	58.8	52.45	55.6	52.75	58	53.05	57.6	53.35	57.2	53.65	56.8	53.95	56.4	54.25	56	54.55	55.6
50.1	54.85	50.3	52.45	50.5	54.25	50.7	53.95	50.9	53.65	51.1	53.35	51.3	53.05	51.5	52.75	51.7	52.45
59	61.85	59	59.45	59	61.25	59	60.95	59	60.65	59	60.35	59	60.05	59	59.75	59	59.45
50.05	53.85	50.15	51.45	50.25	53.25	50.35	52.95	50.45	52.65	50.55	52.35	50.65	52.05	50.75	51.75	50.85	51.45
55.2	59.95	55.6	59.15	56	59.75	56.4	59.65	56.8	59.55	57.2	59.45	57.6	59.35	58	59.25	58.4	59.15
49.05	54.75	49.15	50.75	49.25	53.75	49.35	53.25	49.45	52.75	49.55	52.25	49.65	51.75	49.75	51.25	49.85	50.75
46.1	52.75	46.3	48.75	46.5	51.75	46.7	51.25	46.9	50.75	47.1	50.25	47.3	49.75	47.5	49.25	47.7	48.75

58.05	59	58.15	59	58.25	59	58.35	59	58.45	59	58.55	59	58.65	59	58.75	59	58.85	59
45.1	49.85	45.3	47.45	45.5	49.25	45.7	48.95	45.9	48.65	46.1	48.35	46.3	48.05	46.5	47.75	46.7	47.45
54.05	58.8	54.15	55.6	54.25	58	54.35	57.6	54.45	57.2	54.55	56.8	54.65	56.4	54.75	56	54.85	55.6
41.05	44.85	41.15	42.45	41.25	44.25	41.35	43.95	41.45	43.65	41.55	43.35	41.65	43.05	41.75	42.75	41.85	42.45
46.15	49.95	46.45	49.15	46.75	49.75	47.05	49.65	47.35	49.55	47.65	49.45	47.95	49.35	48.25	49.25	48.55	49.15
56	59.8	56	56.6	56	59	56	58.6	56	58.2	56	57.8	56	57.4	56	57	56	56.6
47	49.85	47	47.45	47	49.25	47	48.95	47	48.65	47	48.35	47	48.05	47	47.75	47	47.45
53.05	56.85	53.15	54.45	53.25	56.25	53.35	55.95	53.45	55.65	53.55	55.35	53.65	55.05	53.75	54.75	53.85	54.45
53	54.9	53	53.3	53	54.5	53	54.3	53	54.1	53	53.9	53	53.7	53	53.5	53	53.3
45.15	49.9	45.45	48.3	45.75	49.5	46.05	49.3	46.35	49.1	46.65	48.9	46.95	48.7	47.25	48.5	47.55	48.3
50.1	54.85	50.3	52.45	50.5	54.25	50.7	53.95	50.9	53.65	51.1	53.35	51.3	53.05	51.5	52.75	51.7	52.45
41.05	44.85	41.15	42.45	41.25	44.25	41.35	43.95	41.45	43.65	41.55	43.35	41.65	43.05	41.75	42.75	41.85	42.45
$y^l_{i(.05)}$	$y^u_{i(.05)}$	$y^l_{i(.15)}$	$y^u_{i(.15)}$	$y^l_{i(.25)}$	$y^u_{i(.25)}$	$y^l_{i(.35)}$	$y^u_{i(.35)}$	$y^l_{i(.45)}$	$y^u_{i(.45)}$	$y^l_{i(.55)}$	$y^u_{i(.55)}$	$y^l_{i(.65)}$	$y^u_{i(.65)}$	$y^l_{i(.75)}$	$y^u_{i(.75)}$	$y^l_{i(.85)}$	$y^u_{i(.85)}$
51	52.9	51	51.3	51	52.5	51	52.3	51	52.1	51	51.9	51	51.7	51	51.5	51	51.3
50.25	55.95	50.75	55.15	51.25	55.75	51.75	55.65	52.25	55.55	52.75	55.45	53.25	55.35	53.75	55.25	54.25	55.15
40.05	41	40.15	41	40.25	41	40.35	41	40.45	41	40.55	41	40.65	41	40.75	41	40.85	41
43.05	46.85	43.15	44.45	43.25	46.25	43.35	45.95	43.45	45.65	43.55	45.35	43.65	45.05	43.75	44.75	43.85	44.45
56.05	59.85	56.15	57.45	56.25	59.25	56.35	58.95	56.45	58.65	56.55	58.35	56.65	58.05	56.75	57.75	56.85	57.45
46	49.8	46	46.6	46	49	46	48.6	46	48.2	46	47.8	46	47.4	46	47	46	46.6
53.25	59.9	53.75	58.3	54.25	59.5	54.75	59.3	55.25	59.1	55.75	58.9	56.25	58.7	56.75	58.5	57.25	58.3
54.05	57.85	54.15	55.45	54.25	57.25	54.35	56.95	54.45	56.65	54.55	56.35	54.65	56.05	54.75	55.75	54.85	55.45
59.05	63.8	59.15	60.6	59.25	63	59.35	62.6	59.45	62.2	59.55	61.8	59.65	61.4	59.75	61	59.85	60.6
45.05	48.85	45.15	46.45	45.25	48.25	45.35	47.95	45.45	47.65	45.55	47.35	45.65	47.05	45.75	46.75	45.85	46.45
55	55.95	55	55.15	55	55.75	55	55.65	55	55.55	55	55.45	55	55.35	55	55.25	55	55.15
40.05	43.85	40.15	41.45	40.25	43.25	40.35	42.95	40.45	42.65	40.55	42.35	40.65	42.05	40.75	41.75	40.85	41.45
49	49.95	49	49.15	49	49.75	49	49.65	49	49.55	49	49.45	49	49.35	49	49.25	49	49.15
39.05	42.85	39.15	40.45	39.25	42.25	39.35	41.95	39.45	41.65	39.55	41.35	39.65	41.05	39.75	40.75	39.85	40.45
42	43.9	42	42.3	42	43.5	42	43.3	42	43.1	42	42.9	42	42.7	42	42.5	42	42.3
51.05	54.85	51.15	52.45	51.25	54.25	51.35	53.95	51.45	53.65	51.55	53.35	51.65	53.05	51.75	52.75	51.85	52.45
47	48.9	47	47.3	47	48.5	47	48.3	47	48.1	47	47.9	47	47.7	47	47.5	47	47.3
50	53.8	50	50.6	50	53	50	52.6	50	52.2	50	51.8	50	51.4	50	51	50	50.6
39.15	43.9	39.45	42.3	39.75	43.5	40.05	43.3	40.35	43.1	40.65	42.9	40.95	42.7	41.25	42.5	41.55	42.3
45.2	49.95	45.6	49.15	46	49.75	46.4	49.65	46.8	49.55	47.2	49.45	47.6	49.35	48	49.25	48.4	49.15
38.15	41.95	38.45	41.15	38.75	41.75	39.05	41.65	39.35	41.55	39.65	41.45	39.95	41.35	40.25	41.25	40.55	41.15
47.05	51.8	47.15	48.6	47.25	51	47.35	50.6	47.45	50.2	47.55	49.8	47.65	49.4	47.75	49	47.85	48.6
57.1	61.85	57.3	59.45	57.5	61.25	57.7	60.95	57.9	60.65	58.1	60.35	58.3	60.05	58.5	59.75	58.7	59.45
55	56.9	55	55.3	55	56.5	55	56.3	55	56.1	55	55.9	55	55.7	55	55.5	55	55.3
56	58.85	56	56.45	56	58.25	56	57.95	56	57.65	56	57.35	56	57.05	56	56.75	56	56.45
39.15	42	39.45	42	39.75	42	40.05	42	40.35	42	40.65	42	40.95	42	41.25	42	41.55	42
48.1	53.8	48.3	50.6	48.5	53	48.7	52.6	48.9	52.2	49.1	51.8	49.3	51.4	49.5	51	49.7	50.6
44.1	49.8	44.3	46.6	44.5	49	44.7	48.6	44.9	48.2	45.1	47.8	45.3	47.4	45.5	47	45.7	46.6
43	44.9	43	43.3	43	44.5	43	44.3	43	44.1	43	43.9	43	43.7	43	43.5	43	43.3
47.05	50.85	47.15	48.45	47.25	50.25	47.35	49.95	47.45	49.65	47.55	49.35	47.65	49.05	47.75	48.75	47.85	48.45
47.25	52	47.75	52	48.25	52	48.75	52	49.25	52	49.75	52	50.25	52	50.75	52	51.25	52
50.2	54.95	50.6	54.15	51	54.75	51.4	54.65	51.8	54.55	52.2	54.45	52.6	54.35	53	54.25	53.4	54.15
41.05	45.8	41.15	42.6	41.25	45	41.35	44.6	41.45	44.2	41.55	43.8	41.65	43.4	41.75	43	41.85	42.6

45.05	47.9	45.15	46.3	45.25	47.5	45.35	47.3	45.45	47.1	45.55	46.9	45.65	46.7	45.75	46.5	45.85	46.3
46.1	48.95	46.3	48.15	46.5	48.75	46.7	48.65	46.9	48.55	47.1	48.45	47.3	48.35	47.5	48.25	47.7	48.15
50	52.85	50	50.45	50	52.25	50	51.95	50	51.65	50	51.35	50	51.05	50	50.75	50	50.45
40.15	43.95	40.45	43.15	40.75	43.75	41.05	43.65	41.35	43.55	41.65	43.45	41.95	43.35	42.25	43.25	42.55	43.15
57.1	62.8	57.3	59.6	57.5	62	57.7	61.6	57.9	61.2	58.1	60.8	58.3	60.4	58.5	60	58.7	59.6
40.15	43	40.45	43	40.75	43	41.05	43	41.35	43	41.65	43	41.95	43	42.25	43	42.55	43
55.1	59.85	55.3	57.45	55.5	59.25	55.7	58.95	55.9	58.65	56.1	58.35	56.3	58.05	56.5	57.75	56.7	57.45
56	59.8	56	56.6	56	59	56	58.6	56	58.2	56	57.8	56	57.4	56	57	56	56.6
35.25	41.9	35.75	40.3	36.25	41.5	36.75	41.3	37.25	41.1	37.75	40.9	38.25	40.7	38.75	40.5	39.25	40.3
$y^l_{i(.05)}$	$y^u_{i(.05)}$	$y^l_{i(.15)}$	$y^u_{i(.15)}$	$y^l_{i(.25)}$	$y^u_{i(.25)}$	$y^l_{i(.35)}$	$y^u_{i(.35)}$	$y^l_{i(.45)}$	$y^u_{i(.45)}$	$y^l_{i(.55)}$	$y^u_{i(.55)}$	$y^l_{i(.65)}$	$y^u_{i(.65)}$	$y^l_{i(.75)}$	$y^u_{i(.75)}$	$y^l_{i(.85)}$	$y^u_{i(.85)}$
57.05	58.95	57.15	58.15	57.25	58.75	57.35	58.65	57.45	58.55	57.55	58.45	57.65	58.35	57.75	58.25	57.85	58.15
90.05	93.85	90.15	91.45	90.25	93.25	90.35	92.95	90.45	92.65	90.55	92.35	90.65	92.05	90.75	91.75	90.85	91.45
29	31.85	29	29.45	29	31.25	29	30.95	29	30.65	29	30.35	29	30.05	29	29.75	29	29.45
75.1	77	75.3	77	75.5	77	75.7	77	75.9	77	76.1	77	76.3	77	76.5	77	76.7	77
35.95	37.85	35.85	35.45	35.75	37.25	35.65	36.95	35.55	36.65	35.45	36.35	35.35	36.05	35.25	35.75	35.15	35.45
94.05	98.8	94.15	95.6	94.25	98	94.35	97.6	94.45	97.2	94.55	96.8	94.65	96.4	94.75	96	94.85	95.6
10.05	12.9	10.15	11.3	10.25	12.5	10.35	12.3	10.45	12.1	10.55	11.9	10.65	11.7	10.75	11.5	10.85	11.3
74.05	76.9	74.15	75.3	74.25	76.5	74.35	76.3	74.45	76.1	74.55	75.9	74.65	75.7	74.75	75.5	74.85	75.3
9	10.9	9	9.3	9	10.5	9	10.3	9	10.1	9	9.9	9	9.7	9	9.5	9	9.3
74.05	77.85	74.15	75.45	74.25	77.25	74.35	76.95	74.45	76.65	74.55	76.35	74.65	76.05	74.75	75.75	74.85	75.45
33.05	37.8	33.15	34.6	33.25	37	33.35	36.6	33.45	36.2	33.55	35.8	33.65	35.4	33.75	35	33.85	34.6
70.05	73.85	70.15	71.45	70.25	73.25	70.35	72.95	70.45	72.65	70.55	72.35	70.65	72.05	70.75	71.75	70.85	71.45
5	7.85	5	5.45	5	7.25	5	6.95	5	6.65	5	6.35	5	6.05	5	5.75	5	5.45
86.1	88.95	86.3	88.15	86.5	88.75	86.7	88.65	86.9	88.55	87.1	88.45	87.3	88.35	87.5	88.25	87.7	88.15
7	8.9	7	7.3	7	8.5	7	8.3	7	8.1	7	7.9	7	7.7	7	7.5	7	7.3
73	77.75	73	73.75	73	76.75	73	76.25	73	75.75	73	75.25	73	74.75	73	74.25	73	73.75
9.05	13.8	9.15	10.6	9.25	13	9.35	12.6	9.45	12.2	9.55	11.8	9.65	11.4	9.75	11	9.85	10.6
70.1	74.85	70.3	72.45	70.5	74.25	70.7	73.95	70.9	73.65	71.1	73.35	71.3	73.05	71.5	72.75	71.7	72.45
5	8.8	5	5.6	5	8	5	7.6	5	7.2	5	6.8	5	6.4	5	6	5	5.6
90.15	96.8	90.45	93.6	90.75	96	91.05	95.6	91.35	95.2	91.65	94.8	91.95	94.4	92.25	94	92.55	93.6
39.05	43.8	39.15	40.6	39.25	43	39.35	42.6	39.45	42.2	39.55	41.8	39.65	41.4	39.75	41	39.85	40.6
86.05	87	86.15	87	86.25	87	86.35	87	86.45	87	86.55	87	86.65	87	86.75	87	86.85	87
13.9	14.85	13.7	12.45	13.5	14.25	13.3	13.95	13.1	13.65	12.9	13.35	12.7	13.05	12.5	12.75	12.3	12.45
96.05	100.8	96.15	97.6	96.25	100	96.35	99.6	96.45	99.2	96.55	98.8	96.65	98.4	96.75	98	96.85	97.6
35.05	37.9	35.15	36.3	35.25	37.5	35.35	37.3	35.45	37.1	35.55	36.9	35.65	36.7	35.75	36.5	35.85	36.3
74	76.85	74	74.45	74	76.25	74	75.95	74	75.65	74	75.35	74	75.05	74	74.75	74	74.45
21.05	24.85	21.15	22.45	21.25	24.25	21.35	23.95	21.45	23.65	21.55	23.35	21.65	23.05	21.75	22.75	21.85	22.45
89.05	92.85	89.15	90.45	89.25	92.25	89.35	91.95	89.45	91.65	89.55	91.35	89.65	91.05	89.75	90.75	89.85	90.45
16.05	19.85	16.15	17.45	16.25	19.25	16.35	18.95	16.45	18.65	16.55	18.35	16.65	18.05	16.75	17.75	16.85	17.45
87.05	89.9	87.15	88.3	87.25	89.5	87.35	89.3	87.45	89.1	87.55	88.9	87.65	88.7	87.75	88.5	87.85	88.3
20	21.9	20	20.3	20	21.5	20	21.3	20	21.1	20	20.9	20	20.7	20	20.5	20	20.3
73.15	76.95	73.45	76.15	73.75	76.75	74.05	76.65	74.35	76.55	74.65	76.45	74.95	76.35	75.25	76.25	75.55	76.15
15.05	17.9	15.15	16.3	15.25	17.5	15.35	17.3	15.45	17.1	15.55	16.9	15.65	16.7	15.75	16.5	15.85	16.3
87.1	92.8	87.3	89.6	87.5	92	87.7	91.6	87.9	91.2	88.1	90.8	88.3	90.4	88.5	90	88.7	89.6
1	1.95	1	1.15	1	1.75	1	1.65	1	1.55	1	1.45	1	1.35	1	1.25	1	1.15
76.1	81.8	76.3	78.6	76.5	81	76.7	80.6	76.9	80.2	77.1	79.8	77.3	79.4	77.5	79	77.7	78.6

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
70.15	73	70.45	73	70.75	73	71.05	73	71.35	73	71.65	73	71.95	73	72.25	73	72.55	73
35	36.9	35	35.3	35	36.5	35	36.3	35	36.1	35	35.9	35	35.7	35	35.5	35	35.3
83	85.85	83	83.45	83	85.25	83	84.95	83	84.65	83	84.35	83	84.05	83	83.75	83	83.45
5	6.9	5	5.3	5	6.5	5	6.3	5	6.1	5	5.9	5	5.7	5	5.5	5	5.3
90.15	96.8	90.45	93.6	90.75	96	91.05	95.6	91.35	95.2	91.65	94.8	91.95	94.4	92.25	94	92.55	93.6
$y^l_{i(.05)}$	$y^u_{i(.05)}$	$y^l_{i(.15)}$	$y^u_{i(.15)}$	$y^l_{i(.25)}$	$y^u_{i(.25)}$	$y^l_{i(.35)}$	$y^u_{i(.35)}$	$y^l_{i(.45)}$	$y^u_{i(.45)}$	$y^l_{i(.55)}$	$y^u_{i(.55)}$	$y^l_{i(.65)}$	$y^u_{i(.65)}$	$y^l_{i(.75)}$	$y^u_{i(.75)}$	$y^l_{i(.85)}$	$y^u_{i(.85)}$
26	29.8	26	26.6	26	29	26	28.6	26	28.2	26	27.8	26	27.4	26	27	26	26.6
72.15	76.9	72.45	75.3	72.75	76.5	73.05	76.3	73.35	76.1	73.65	75.9	73.95	75.7	74.25	75.5	74.55	75.3
20	23.8	20	20.6	20	23	20	22.6	20	22.2	20	21.8	20	21.4	20	21	20	20.6
91.2	95	91.6	95	92	95	92.4	95	92.8	95	93.2	95	93.6	95	94	95	94.4	95
26.05	28.9	26.15	27.3	26.25	28.5	26.35	28.3	26.45	28.1	26.55	27.9	26.65	27.7	26.75	27.5	26.85	27.3
61.1	65.85	61.3	63.45	61.5	65.25	61.7	64.95	61.9	64.65	62.1	64.35	62.3	64.05	62.5	63.75	62.7	63.45
13	17.75	13	13.75	13	16.75	13	16.25	13	15.75	13	15.25	13	14.75	13	14.25	13	13.75
73.1	76.9	73.3	75.3	73.5	76.5	73.7	76.3	73.9	76.1	74.1	75.9	74.3	75.7	74.5	75.5	74.7	75.3
10	13.8	10	10.6	10	13	10	12.6	10	12.2	10	11.8	10	11.4	10	11	10	10.6
91.05	93.9	91.15	92.3	91.25	93.5	91.35	93.3	91.45	93.1	91.55	92.9	91.65	92.7	91.75	92.5	91.85	92.3
12.05	15.85	12.15	13.45	12.25	15.25	12.35	14.95	12.45	14.65	12.55	14.35	12.65	14.05	12.75	13.75	12.85	13.45
84.1	89.8	84.3	86.6	84.5	89	84.7	88.6	84.9	88.2	85.1	87.8	85.3	87.4	85.5	87	85.7	86.6
14.05	16.9	14.15	15.3	14.25	16.5	14.35	16.3	14.45	16.1	14.55	15.9	14.65	15.7	14.75	15.5	14.85	15.3
68.05	71.85	68.15	69.45	68.25	71.25	68.35	70.95	68.45	70.65	68.55	70.35	68.65	70.05	68.75	69.75	68.85	69.45
13.05	16.85	13.15	14.45	13.25	16.25	13.35	15.95	13.45	15.65	13.55	15.35	13.65	15.05	13.75	14.75	13.85	14.45
86.2	91.9	86.6	90.3	87	91.5	87.4	91.3	87.8	91.1	88.2	90.9	88.6	90.7	89	90.5	89.4	90.3
30.1	32.95	30.3	32.15	30.5	32.75	30.7	32.65	30.9	32.55	31.1	32.45	31.3	32.35	31.5	32.25	31.7	32.15
86	88.85	86	86.45	86	88.25	86	87.95	86	87.65	86	87.35	86	87.05	86	86.75	86	86.45
14.05	18.8	14.15	15.6	14.25	18	14.35	17.6	14.45	17.2	14.55	16.8	14.65	16.4	14.75	16	14.85	15.6
87.05	89.9	87.15	88.3	87.25	89.5	87.35	89.3	87.45	89.1	87.55	88.9	87.65	88.7	87.75	88.5	87.85	88.3
39	39.95	39	39.15	39	39.75	39	39.65	39	39.55	39	39.45	39	39.35	39	39.25	39	39.15
94.15	99.85	94.45	97.45	94.75	99.25	95.05	98.95	95.35	98.65	95.65	98.35	95.95	98.05	96.25	97.75	96.55	97.45
23.05	27.8	23.15	24.6	23.25	27	23.35	26.6	23.45	26.2	23.55	25.8	23.65	25.4	23.75	25	23.85	24.6
68	71.8	68	68.6	68	71	68	70.6	68	70.2	68	69.8	68	69.4	68	69	68	68.6
15.1	19.85	15.3	17.45	15.5	19.25	15.7	18.95	15.9	18.65	16.1	18.35	16.3	18.05	16.5	17.75	16.7	17.45
83.1	88.8	83.3	85.6	83.5	88	83.7	87.6	83.9	87.2	84.1	86.8	84.3	86.4	84.5	86	84.7	85.6
22.05	27.75	22.15	23.75	22.25	26.75	22.35	26.25	22.45	25.75	22.55	25.25	22.65	24.75	22.75	24.25	22.85	23.75
66.15	69.95	66.45	69.15	66.75	69.75	67.05	69.65	67.35	69.55	67.65	69.45	67.95	69.35	68.25	69.25	68.55	69.15
8	8.95	8	8.15	8	8.75	8	8.65	8	8.55	8	8.45	8	8.35	8	8.25	8	8.15
90.05	93.85	90.15	91.45	90.25	93.25	90.35	92.95	90.45	92.65	90.55	92.35	90.65	92.05	90.75	91.75	90.85	91.45
16	19.8	16	16.6	16	19	16	18.6	16	18.2	16	17.8	16	17.4	16	17	16	16.6
75.2	79	75.6	79	76	79	76.4	79	76.8	79	77.2	79	77.6	79	78	79	78.4	79
24.2	29.9	24.6	28.3	25	29.5	25.4	29.3	25.8	29.1	26.2	28.9	26.6	28.7	27	28.5	27.4	28.3
73.05	78.75	73.15	74.75	73.25	77.75	73.35	77.25	73.45	76.75	73.55	76.25	73.65	75.75	73.75	75.25	73.85	74.75
17.05	21.8	17.15	18.6	17.25	21	17.35	20.6	17.45	20.2	17.55	19.8	17.65	19.4	17.75	19	17.85	18.6
80.15	87.5	80.45	83.75	80.75	86.75	81.05	86.25	81.35	85.75	81.65	85.25	81.95	84.75	82.25	84.25	82.55	83.75