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An Optimal Replenishment Policy for the EPQ model with Permissible Delay in Payments

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Abstract. Most of the past researches on economic replenishment quantity did not consider the trade credit period. In practice, however, companies in Taiwan have shown that taking the trade credit period into account when they are making inventory decisions decreases costs substantially. The aim of this paper is to develop an inventory model that incorporates a delay in payments to minimize the total inventory-relevant cost. The optimal replenishment policy of the proposed model is identified by utilizing the global optimum conditions and non-constrained quadratic nonlinear programming. A numerical example is also provided.

Keywords: EPQ, Optimal Replenishment Policy, Delay in Payments

1. Introduction

Inventory is one of the most important assessments when a firm attempts to improve the efficiency of the operations in its supply chain. Inventory control, which is linked to most aspects of production organization, can influence a company's business and has a critical role in the activities of operations management. Therefore, a company can take advantage of tracking stock by shortening the lead time for replenishment and estimating inventory cost as long as the company maintains a sturdy inventory management system.

Goyal (1985) argued that IBM's efforts in integrating their national spare part stock network, developing a new inventory system, and modifying their customer service quality have reduced their total inventory cost by more than $250,000,000 and continuously saves the company up to $20,000,000 in annual expenses. These findings demonstrate the importance of inventory management in a company's business. High-working capital on inventory could result in expenditures such as holding costs and insurance, and the costs of other personnel matters might dramatically rise. Most manufacturers believe that flawlessly managing their inventory can not only reduce inventory cost but also increase their freedom when they use the capital in their companies.

In reviewing the literature on inventory management, researcher like Wilson (1934) extended the original model and applied it to solve inventory problems in the real world. In addition, most Economic Production Quantity (EPQ) models incorporate the following assumptions:

- Production rate must be greater than demand
- There must be instantaneous replenishment

In fact, the order submitted by retailer usually limits the production problems that supplier need to deal with. Therefore, it is necessary to relax the EPQ assumptions to create an inventory model that will find a better solution for the issues that suppliers face.

Most of the traditional EPQ models assume that retailers pay for the goods immediately when the transaction is completed. In reality, few retailers pay for their goods immediately because most of them use trade credit. The use of trade credit violates the EPQ assumptions. Furthermore, to create a win-win situation, both the supplier and the retailer usually set up a reasonable credit transaction deadline by negotiating with each other. By setting a deadline, the supplier will accrue extra interest, and the retailer will not need to initiate loan financing. When delayed payment is provided by suppliers, the retailer also has the advantage of reducing interest yielded by the fund which uses in the payment of backlog.

Based on the reasons we mention above, we construct a theoretical model that incorporates a delay in payments under conditions that does not allow shortages. We then demonstrate some important properties associate with implementing an optimal replenishment policy. After obtaining the model properties, we generate an example. In the last section of the paper, we provide our conclusions as well as suggestions for further research.

2. Literature Review

Goyal (1985) suggested an EOQ model with a fixed credit deadline for paying for goods as a promotion that could be provided by the supplier. If the retailer sells the product within the limit, he/she will be spared the interest generated by the funds that must be used to pay for the goods. That is a type of chance cost. If the pay date passes, he/she must pay for the products and the interest generated by the funds that have already been paid to the supplier for the rest of the products that are in stock. Many researchers have extended the above model in different ways.

Chung (1989) investigated the interactions between credit transactions and the time value of currency, and he then employed a discounted cash flow (DCF) model to build a total variant cost function for inventory. In his research, an optimal replenishment policy was provided. Chung (1998) revised the model by Goyal (1985) and also found an optimal order quantity. Huang (2004, 2007) rewrote the assumptions made by Goyal (1985) and argued that the selling price and the purchase cost for the product might not be equal in real-world scenarios. Huang also considered the rate of replenishment to be finite and developed his research in two phases: one where the supplier allows the retailer to pay for the products before the credit deadline, and another phase where the retailer also provides his/her customers with permission to delay payments for the sake of increasing the amount of product that is sold. Huang (2007) also discussed a possibility in which the supplier offers partial permission for delays in payment when the actual order quantity is less than a certain amount. He minimized the total cost to ensure that the inventory cycle and the order quantity were optimal.

Liao (2007) investigated a non-instantaneous inventory model of deterioration for items that incorporated delays in payment. Tsao and Sheen (2008) found an optimal selling price and maximized the profit for deterioration items while including permission for delays in payment. Chang, Teng and Goyal (2008) reviewed the literature for inventory models published in the last twenty years and classified these models as non-deterioration items, deterioration items, models that allow shortages, order quantity and inflation. They also suggested further research directions for every model they reviewed.

Chung (2009) constructed a solution procedure for non-instantaneous deteriorating items with permissible delay in payment. Ouyang, Teng, Goyal and Yang (2009) extended the work by Goyal (1985) by making a rule that the promotion cannot be offered if the order quantity submitted by the retailer is below a certain threshold. Chung and Huang (2009) extended the work of Goyal (1985) by using dual variables and the year relevant cost to construct an inventory model. They also proved that the year relevant cost function was convex. A year later, Hu and Liu (2010) adopted the finite replenishment rate assumption and relaxed a condition in which purchase price and selling price must not be consistent as part of their effort to build upon the work of Chung and Huang (2009). Furthermore, they found an optimal inventory policy in the environment they had made. Chang (2010) derived an optimal replenishment policy with stock-dependent demand. Maihami (2012) derived an optimal replenishment policy for non-instantaneous deteriorating items under permissible delay in payments and partial backlogging. In his research, item price is also controlled.

3. The Model

3.1. Notations

D	Demand rate
P	Production rate
h	Holding cost per unit of product in stock
I_{max}	Maximum inventory level
t_1	The time point when the quantity of products in stock reaches I_{max}
T	Inventory cycle
s	Sale price
c	Purchase price
I_m	Inventory level at the time point of $t = M$
I_e	Interest income for the unit fund
I_k	Interest yielded by the unit fund, which is used to pay for the product in stock.
M	Credit transaction deadline
$TVC(T)$	Total cost function

3.2. Assumptions

A3.1 Single product with non-deteriorating properties

A3.2 If both D and P are known and constant, it is reasonable to assume that $P > D$

A3.3 Purchase price and sale price of the product are both constant and $s > c$

A3.4 Instant replenishment

A3.5 $I_k \geq I_e$

A3.6 When $0 \leq T \leq M$, the retailer does not have to pay for the interest, I_e, for the products in stock. Otherwise, when $0 \leq M \leq T$, the retailer needs to pay the interest, I_k.

3.3. Model Formulation

Based on the above notation and assumptions, the inventory model is developed and is presented in Figure 1.

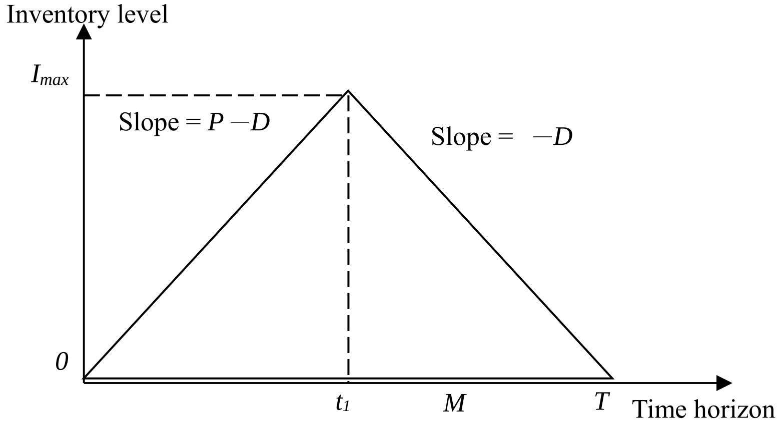

Fig. 1. The inventory level during $0 \leq M \leq T$

We then investigate the optimal replenishment policy under the conditions of $0 \leq M \leq T$ and $0 \leq T \leq M$, respectively.

The objective of the model is to minimize the total cost. Thus, we first define the total cost as equal to the sum of the holding cost, the interest yielded by the funds used to pay for the product in stock and the negative interest income. By using some triangular properties, we find the formulas of $t_1 = \dfrac{DT}{P}$ and $I_{\max} = DT(1 - \dfrac{D}{P})$ for computing other cost functions and determining the properties of an optimal policy.

The holding cost is:

$$h\left[\int_0^T I(t)dt\right] = \frac{hDT^2}{2}(1 - \frac{D}{P}) \tag{1}$$

Case I: $0 \leq t_1 \leq M \leq T$

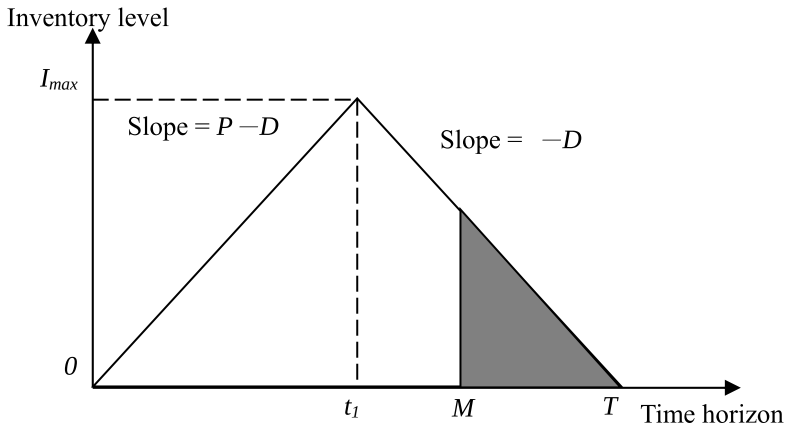

Fig. 2. The interest yielded by the unit fund, which is used to pay for the product in stock during $0 \le M \le T$

The interest yielded by the unit fund, which is used to pay for the product in stock during inventory cycle T is:

$$cI_k \int_M^T I(t)dt = \frac{cI_k D(T-M)^2}{2} \tag{2}$$

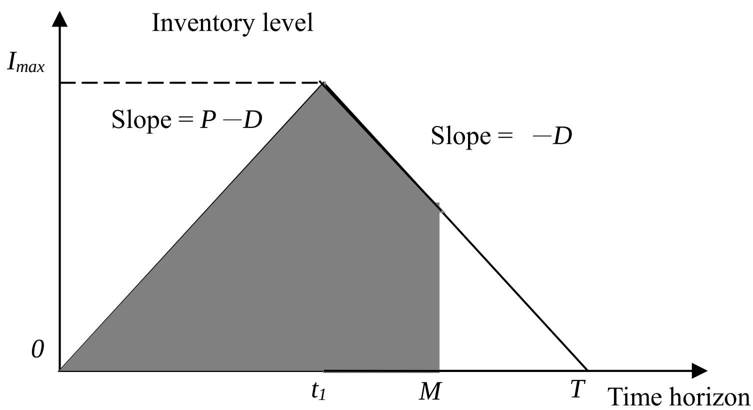

Fig. 3. The interest income during $0 \le M \le T$

It is reasonable to assume that interest income is the sum of chance cost yielded by the unpaid payments of the products and the interest yielded by the income from selling products, that is

$$cI_e \int_0^M Pdt + sI_e \int_0^T Ddt = I_e(cPM + sDT) \tag{3}$$

Case II: $0 \leq T \leq M$

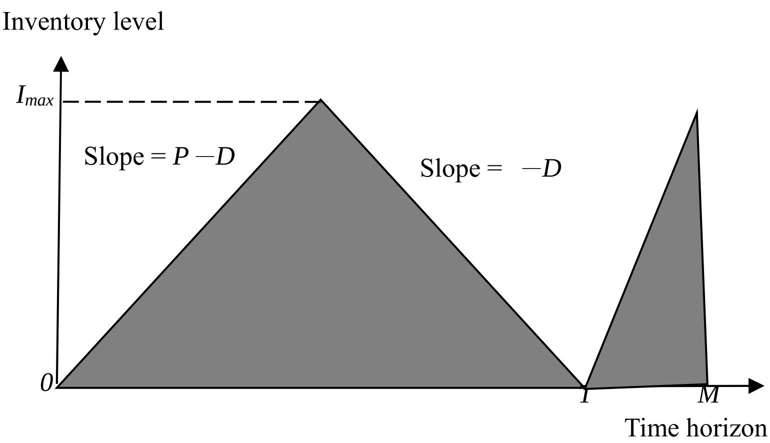

Fig. 4. The interest income during $0 \leq T \leq M$

Under this condition, retailers sell all of the products in stock before they have to pay for the products they purchased. Therefore, they do not have to pay interest.

From Figure 4, we construct the function of interest income as follows.

$$cI_e \int_0^{t_1} P dt + sI_e \int_0^{T} D dt = I_e DT(c+s) \tag{4}$$

Furthermore, we set up the following total cost, TVC(T) , according to the length of credit transaction deadline.

$$TVC(T) = \begin{cases} TVC_1(T), & 0 \leq t_1 \leq M \leq T \\ TVC_2(T), & 0 \leq T \leq M \end{cases}$$

For the two different length of credit limits, the total cost functions can be constructed as follows:

When $0 \leq t_1 \leq M \leq T$,

$$TVC_1(T) = \frac{hDT^2}{2}(1 - \frac{D}{P}) + \frac{cI_k D}{2}(T - M)^2 - I_e DT(c+s) \tag{5}$$

and when $0 \leq T \leq M$,

$$TVC_2(T) = \frac{hDT^2}{2}(1 - \frac{D}{P}) - I_e DT(c+s) \tag{6}$$

3.4. Properties

We then take the first order derivatives of TVC1(T) and TVC2(T) with respect to inventory cycle, T, respectively, and let them equal to 0. By solving the equation, we obtain the optimal replenishment time.

$$\frac{dTVC_1(T)}{dT} = hDT(1-\frac{D}{P}) + cDI_k(T-M) - DI_e(c+s) \tag{7}$$

$$T_1^* = \frac{(cI_kM + I_e(c+s))P}{h(P-D)+cI_kP} \tag{8}$$

To assure that T_1^* is the global minimum, we take the derivative of (7) with respect to T again.

$$\frac{d^2TVC_1(T)}{d^2T} = hDT(1-\frac{D}{P}) - I_eD(c+s) \tag{9}$$

Lemma 1 : When 0≤M≤T, the sufficient condition under which TVC_1(T) is convex is $T > \frac{PI_e(c+s)}{h(P-D)}$

Proof: The proof is a straightforward consequence of (9) if we set a suitable value, T', that makes $T' > \frac{PI_e(c+s)}{h(P-D)}$ and substitute it into (9).

Denote T_1^* as the extreme point of TVC_1(T), then, we have the following solutions.

$$TVC_1'(T) \begin{cases} <0\,,\ T \in (0, T_1^*) \\ =0\,,\ T = T_1^* \\ >0\,, T \in (T_1^*, \infty) \end{cases} \tag{10}$$

By repeating the procedure of finding T_1^*, we attempt to find the derivates of $TVC_2(t_2, T)$ and T_2^*.

By taking the first order derivative of $TVC_2(T)$ with respect to T, we obtain

$$\frac{dTVC_2(T)}{dT} = hDT(1-\frac{D}{P}) + I_e D(c+s) \tag{11}$$

When (11) equals 0, we obtain the optimal replenishment time after some rearranging.

$$T_2^* = \frac{I_e P(c+s)}{h(P-D)} \tag{12}$$

Now, to examine the convexity of $TVC_2(T)$, we take the second order derivative of $TVC_2(T)$ with respect to T.

$$\frac{d^2 TVC_2(T)}{d^2 T} = hD(1-\frac{D}{P}) \tag{13}$$

Denote T_2^* as the extreme point of $TVC_2(T)$, then we have the following solutions.

$$TVC_2'(T) \begin{cases} <0, & T \in (0, T_2^*) \\ =0, & T = T_2^* \\ >0, & T \in (T_2^*, \infty) \end{cases} \tag{14}$$

Lemma 2: TVC_2(T) is a convex function during 0≤M≤T.

Proof: From A3.3, it is clearly that (13) is greater than 0. Therefore, second order condition shows the result.

4. Numerical Example

To obtain sufficient usage of their production capacity, suppliers will offer retailers a promotion for delaying payments until a credit transaction deadline, M. Therefore, the retailers can apply the results found in this paper to build their inventory model. Furthermore, an optimal replenishment policy that dramatically reduces total cost can also be obtained. Data were collected from some companies located in Taiwan and processed with the model. To mimic reality, the parameters used in the proposed model are assigned as follows:

$P = 50 \quad D = 30 \quad h = 0.1 \quad c = 1$

$M = 1.2 \quad s = 8 \quad I_e = 0.01 \quad I_k = 0.12$

After performing the calculations, we obtain $T_1^* = 1.4625$ and $T_2^* = 1.2$. When $M > t_2$, we find that the longer the optimal inventory cycle is, the greater the profit is that retailers could obtain.

As a result, we plot TVC_1 and TVC_2 versus T in Figures 5 and 6, respectively. These two figures provide evidence that the aforementioned lemmas are valid. In addition, the delay in payments offered by certain suppliers would result in a negative inventory cost, which also leads to more products being purchased from the suppliers.

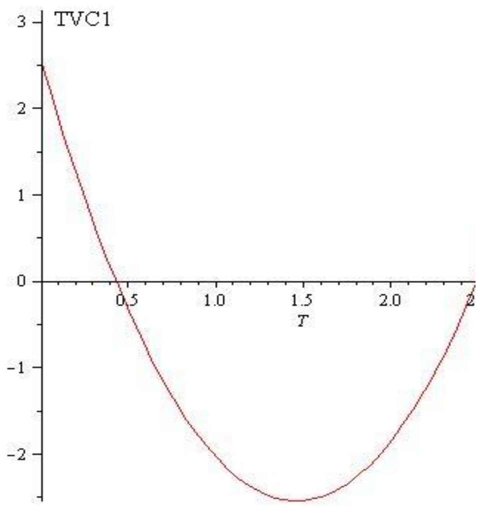

Fig. 5. The value of TVC_1 versus T when $0 \leq t_1 \leq M \leq T$

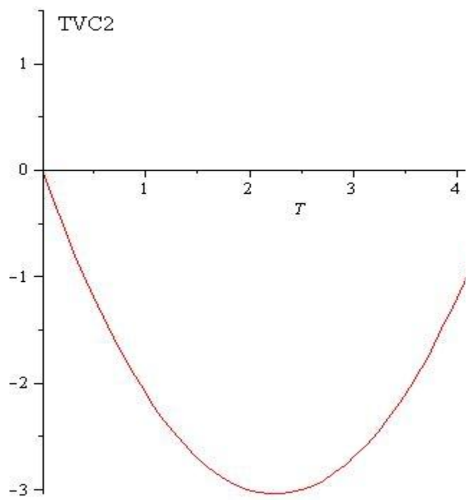

Fig. 6. The value of TVC_2 versus T when $0 \leq T \leq M$

5. Conclusions

The aim of this paper is to develop an inventory model that incorporates delays in payments for impermissible shortages. The proposed model for an optimal replenishment policy is identified by utilizing the global optimal conditions for the Maple-based numerical analysis. We find that both TVC_1(T) and TVC_2(T) are convex functions. This result assures the existence of the polar point that is unique under certain conditions. Moreover, as shown in Figures 5 and 6, adopting the promotion offered by the suppliers would benefit the retailers' businesses because the retailers would be more willing to purchase products from the suppliers. Therefore, a win-win scenario is achieved.

Accordingly, in this research, we assume that the product is not deteriorating and that no defective items are produced during the replenishment process. For future analyses, we will relax the assumptions by considering deteriorating items and assuming that the demand rate and replenishment rate follow certain probability distributions (e.g., a Weibull distribution or GAMMA distribution) to provide solutions for replenishment issues in the real world. In addition, taking the factors of inflection and time preference into account would also be worth investigating.

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