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Model and Algorithm of Reliable Location-Routing Problem for Perishable Goods

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Abstract. Based on the characteristics of perishable goods, we extend the location-routing problem with distance constraints by adopting chance constrained goal programming approach to solve the uncertainty and to express the different priority levels of decisions. The random of depot construction costs and vehicle routing travel time is taken into account. Meanwhile, a two-stage genetic algorithm incorporated with stochastic simulation is employed to solve the model. Computational experience is presented to illustrate the effectiveness of the solution procedure.

Keywords: Location Routing, Perishable Goods, Stochastic Programming, Two-stage Genetic Algorithm

1. Introduction

The purpose of this paper is to address the reliable location-routing problem for perishable goods. Examples of perishable goods involve food products, vegetables, flowers, living animals and ready-mix concrete etc (Chen et al., 2009).

After extensive field observations, we chose to focus on two aspects that seem important to perishable goods supply chain design: the depot facility location and vehicle routing. Besides, the location of depots and the routing of vehicles cannot be treated separately. Baldacci et al.(2011),Salhi and Rand(1989) evaluate the effect of ignoring routing when locating facilities and clearly show that separating facility location from vehicle routing may lead to suboptimal decisions. This interdependence between the depots location and the vehicle routing leads to Location-Routing Problems (LRP)(Toyoglu et al.,2012). Due to its significant practical consequence and theoretical value, the LRP has been extensively studied in the past few decades. Detailed reviews of the literature of LRP are given in Nagy and Salhi(2007),Lopes et al.(2013).

For perishable goods, the particular characteristic is that the vehicle routes are constrained to be short and the vehicle does not have to return to the original depot within the time window. Berger et al.(2007) formulate a location-routing problem with distance constraints (LRP-DC), a variant of a fixed-charge facility location problem (FLP), which is a special and uncomplicated formulation of LRPs.

Uncertainty is the basic issue which should be taken into account in designing reliable LRP problem. Two essential elements, the construction costs of depot facilities and the travel time of vehicles, are hard to predict exactly in advance. Classical location-routing models treat data as though they were known and deterministic, yet ignoring data uncertainty can result in highly sub-optimal solutions. Snyder(2006) reviews the uncertainty modelling method in facility location problem. There are two principal approaches can be used to formulate reliable mathematical models which are able to describe uncertain conditions: (a) robust optimization (RO), (b) stochastic programming (SP). Robust optimization approach often attempts to optimize the worst-case performance of the system (Hong et al.,2012). Stochastic programming models are similar in style but try to take advantage of the fact that probability distributions governing the data are known or can be estimated(Shapiro et al.,2009). The chance constrained programming(CCP) relaxes the constraints in deterministic mathematical programming and replaces them with probabilistic constraints, where some or all data elements are random and the constraints are required to hold with at least some level of reliability(Liu,2009).

To the authors' best knowledge, reliability analysis with multi-objectives in conflict has received little attention in the study of location-routing problem for perishable goods in spite of its importance in the past. This paper focuses on LRP in uncertainties by adopting chance constrained goal programming framework and the development of a two-stage genetic algorithm incorporated with stochastic simulation for solving the stochastic LRP models. Computational result demonstrates the validity of the proposed method.

The remainder of this paper is organized as follows: in the next section, the model for location-routing problem with chance constrained goal programming framework is presented and analyzed. These are followed by a hybrid intelligent algorithm used in the problem. Following these, the heuristic method is tested using computational experimental data. We finish with some conclusion remarks in Section 5.

2. Proposed chance-constrained goal programming model

2.1. Notation

Sets

I =the set of demand locations

J =the set of candidate facility locations

$G=(N,A)$ =the graph

$N = I \cup J$ =the set of nodes

$A = N \times N$ =the set of arcs

P_j =the set of all feasible routes associated with facility $j, \forall j \in J$

L =the set of goals

Parameters

d_{ij} =the distance between node i and j, $\forall (i,j) \in A$

k =the feasible route associated with facility j

$$a_{ijk} = \begin{cases} 1 & \text{if route } k \text{ associated with facility } j \text{ visits customer } i, \forall i \in I, \forall j \in J, \forall k \in P_j \\ 0 & \text{otherwises} \end{cases}$$

c_{jk} =cost of route k associated with facility $j, \forall j \in J$

f_j =fixed construction cost associated with selecting facility $j, \forall j \in J$

α =objective weighting factor

$d_l^+ \vee 0$ =the positive deviation from the target of goal l

$d_l^- \vee 0$ =the negative deviation from the target of goal l

ξ_j =the stochastic construction cost variable associated with selecting facility $j, \forall j \in J$

τ_{jk} =the stochastic cost of route k associated with facility $j, \forall j \in J$

Decision Variables

$$X_j = \begin{cases} 1 & \text{if facility } j \text{ is selected, } \forall j \in J \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{jk} = \begin{cases} 1 & \text{if route } k \text{ associated with facility } j \text{ is selected,} \forall i \in I, \forall j \in J, \forall k \in P_j \\ 0 & \text{otherwises} \end{cases}$$

2.2. Chance-constrained goal programming formulation

Liu(2009) integrates Chance-Constrained Programming(CCP) with Goal program- ming(GP), which is called Chance-Constraint Goal Programming(CCGP), to deal with the stochastic constraints with multi-objectives in conflict.

The location-routing model with CCGP can be formulated as follows:

$$\text{lexmin } \left\{ d_1^- \vee 0, d_2^- \vee 0 \right\} \tag{1}$$

Subject to:

$$\Pr\left\{ \sum_{j \in J} \xi_j X_j \le B \right\} - \beta = d_1^- \tag{2}$$

$$\alpha - \Pr\left\{ \tau_{jk} Y_{jk} \le T \right\} = d_2^-, \quad \forall j \in J, \forall k \in P_j \tag{3}$$

$$\sum_{j \in J} X_j \le P \tag{4}$$

$$\sum_{j \in J} \sum_{k \in P_j} a_{ijk} Y_{jk} = 1, \quad \forall i \in I \tag{5}$$

$$X_j - Y_{jk} \ge 0, \quad \forall j \in J, \forall k \in P_j \tag{6}$$

$$X_j \in \left\{0,1\right\}, \quad \forall j \in J \tag{7}$$

$$Y_{jk} \in \left\{0,1\right\}, \quad \forall j \in J, \forall k \in P_j \tag{8}$$

Lexmin in the objective function (1) represents lexicographically minimizing the objective vector, which includes the negative deviation from the two goals with priority level, the fixed depot facility location costs and the routing cots to the customers. Constraint (2) and (3) are the goal equations. Constraint (4) states that no more than P facilities are to be located. Constraint (5) requires each demand node to be on one route. Constraint (6) imposes that a route can be assigned only to an open facility. Constraint (7) and (8) are standard binary restrictions on the variables.

3. Method of Solution

In general, the LRP is NP-hard since they merge two NP-hard problems: depot facility location and vehicle routing. It is difficult to solve by traditional calculus-based optimization methods. Almost all surveys urge the use of heuristics due to the complexity of LRPs.

In this problem, we design a solution procedure consisting of a two-stage genetic algorithm (GA) and Monte-Carlo simulation to handle the model. The upper level GA is employed to solve the location-routing problem, of which the fitness function is corresponding to the construction cost of the facilities and lower best fitness, while the lower level GA is used to solve the routing problem, of which the fitness is relative to total travel time. The lower level GA gives its feedback on the upper level GA.

The Monte Carlo simulation is used to compute the uncertain functions in the model. The simulation is based on sampling random variables from probability distributions (Liu,2009).

3.1. Computing optimal designs

3.1.1. Representation of decision variables

The representation of the decision variables in GA fashion, namely a chromosome, is an important aspect of applying GA. For the LRP studied here, the variables are codes as follows.

Location variable:

x_j is coded as binary array, 1 for setting facility at node j and 0 for otherwise.

Routing variable:

We note that the operational plan is fully determined by the decision vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$.

Vector $\mathbf{a} = (a_1, a_2, \cdots, a_n)$: integer decision vector representing n customers with $1 \le a_1 \le n$ and $a_i \ne a_j$ for all $i \ne j, i, j = 1, 2, \ldots, n$.

Vector $\mathbf{b} = (b_1, b_2, \cdots, b_{v-1})$: integer decision vector with $b_0 \equiv 0 \le b_1 \le b_2 \le \cdots \le b_{v-1} \le n \equiv b_v$, v for number of vehicles.

Vector $\mathbf{c} = (c_1, c_2, \cdots, c_{p-1})$: integer decision vector with $c_0 \equiv 0 \le c_1 \le c_2 \le \cdots \le c_{p-1} \le v \equiv c_p$, p for number of facilities.

If $b_l = b_{l-1} (1 \le l \le m-1)$, then vehicle l is not used, and if $b_l > b_{l-1}$ then k^{th} vehicle l is used.

When k^{th} vehicle used, if $k < c_q$, then the route of vehicle $k(1 \le k \le v-1)$: facility $c_q (1 \le q \le p-1) \to a_{b_{k-1}+1} \to a_{b_{k-1}+2} \to \cdots \to a_{b_k} \to c_q$,and if $l \ge c_q$,then compare l and c_{q+1}.

	a_1 a_2		a_3		a_3	
$b_0 \equiv 0$	$b_1=2$ $b_2=2$	$b_3 \equiv 4$
$c_0 \equiv 0$	$c_1=2$	$c_2 \equiv 2$

Fig. 1: An Example of representation of decision variables

For example, there are 4 customers, 3 vehicles, and 2 facilities. The routing variables are displayed in Fig.1.In this Figure, Route 1: Facility $1 \to a_1 \to a_2 \to$ Facility 1, Route 2: Facility $2 \to a_3 \to$ Facility 2, Route 3: Facility $2 \to a_4 \to$ Facility 2.

3.1.2. Main framework of the optimal algorithm

The main view of the algorithm is described as follows:

Step1: Initialization. Coding location variables with binary array, and generating the upper initial population, and set iteration counter $i = 0$.

Step2: Fitness calculation. Calculating the fitness of each upper chromosome.

Step2.1: Initialization. Coding routing variables with integer number array, and generating the lower initial population, and set iteration counter $j = 0$.

Step2.2: Fitness calculation. Calculating the fitness of each lower chromosome.

Step2.3: Genetic Operations. Crossover and mutation.

Step2.4: Convergence check. If iteration counter j is equal to generations limit, output best result, else $j = j + 1$, go to Step 2.2.

Step3: Genetic Operations. Crossover and mutation.

Step4: Convergence check. If iteration counter is equal to generations limit, output best result, else $i = i + 1$, go to Step 2.

The flow diagram for a stochastic simulation-based GA for LRP is shown in Fig.2.

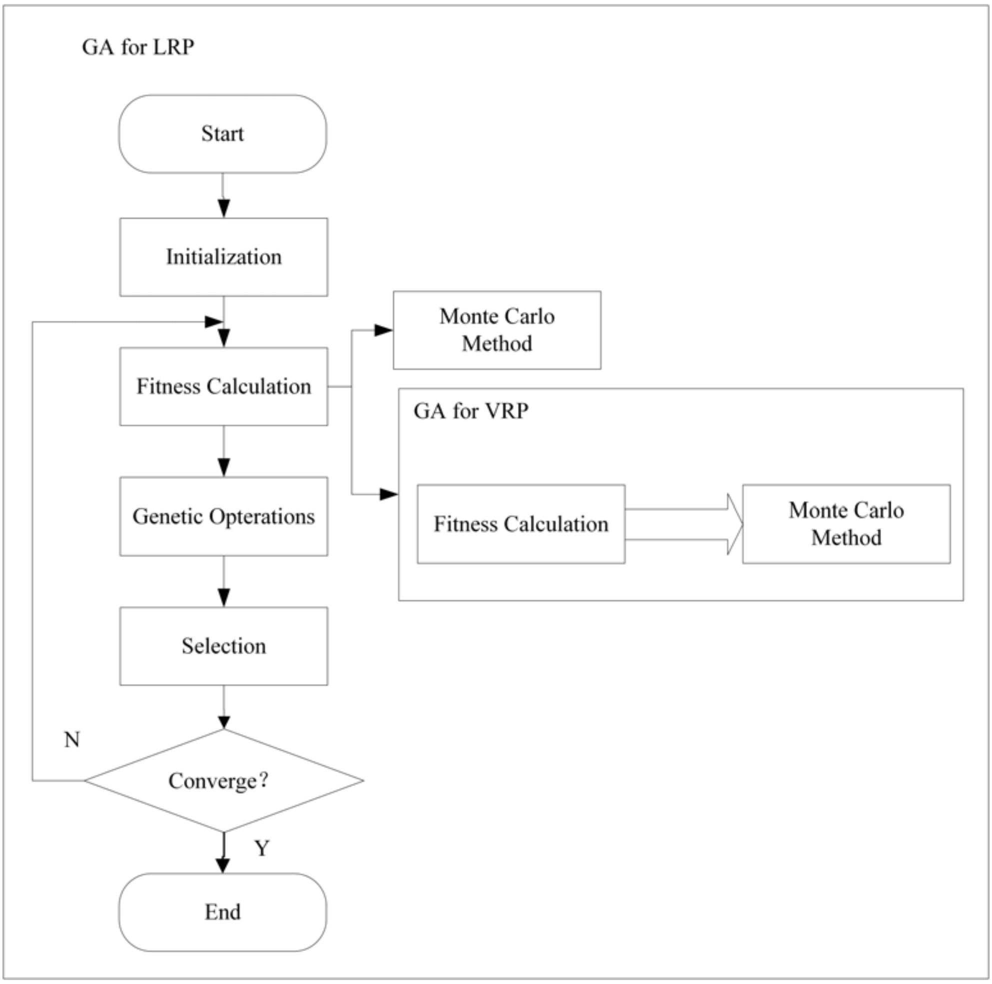

Fig. 2: An Example of representation of decision variables

4. Numerical Example

In this section, we use a case study to evaluate the performance of presented model and algorithm. The network, given by Daskin(1995), is depicted in Fig.3. The network consists of 12 nodes, 18 links.

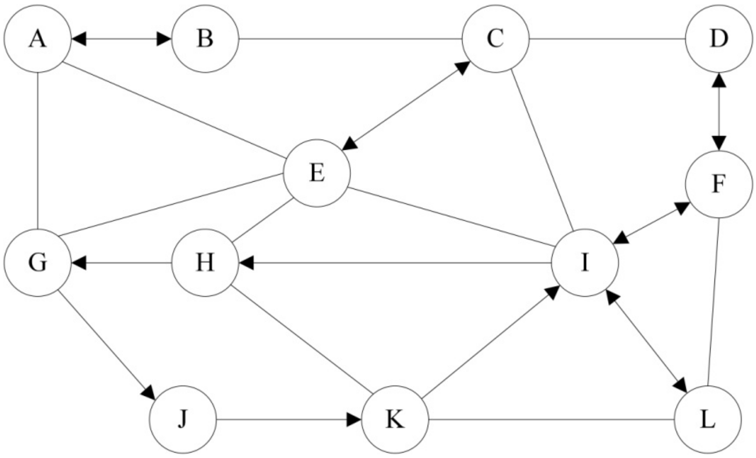

Fig. 3: Sample network and its optimal locations

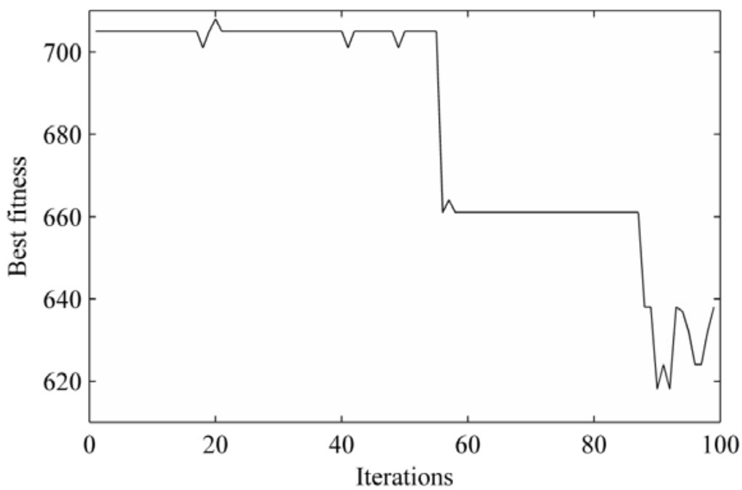

Fig. 4: Convergence curve of best objective

The algorithm described above is implemented in C# language. Console output allows users to monitor the iteration process. In this example, we set $\alpha = 0.5, \beta = 0.5$, working as present constant variables. The CPU searching time, on Intel (R) Core (TM) 2 Duo T6400 2.0GHz, RAM 2GB, is 288 seconds. When we set number limit of facilities = 3, number limit of routes =5, the optimal depot facility location solution is {B, E, I}, and the final best fitness is

638. The convergence curve of best objective and total objective are shown in Fig.4. and Fig.5.

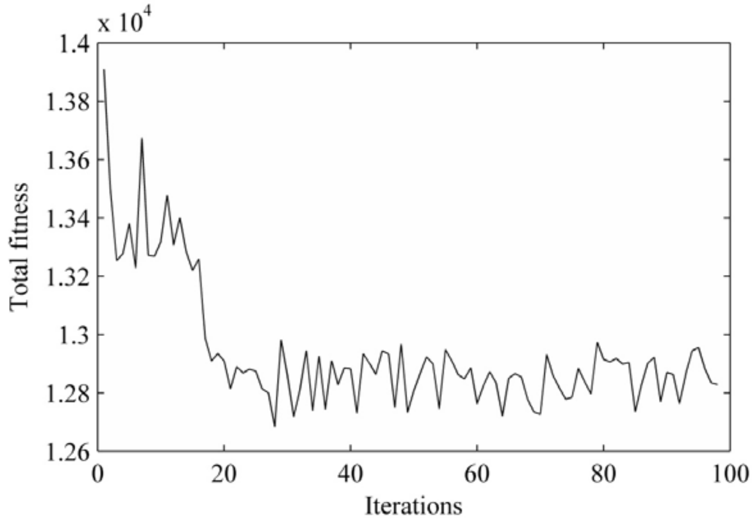

Fig. 5: Convergence curve of total objective

5. Conclusion

This paper is concerned with the reliable location-routing problem for perishable goods. According to the characteristic of perishable goods, we formulate a Chance-Constrained Goal Programming framework to optimize depot locations and vehicle routing. The deterministic parameters are replaced by the probability one. And the hierarchical multi-objective is addressed by goal programming. Through CCGP, the priority goal can be directly calculated into the optimization process and uncertainties in the model's coefficients which are expressed as stochastic variables, greatly enhancing the robustness of the optimization system. The model is then applied to a case study and solved through a two-step genetic algorithm incorporating Monte-Carlo simulation. Results from the experiment suggest that the proposed LRP- CCGP model is applicable to perishable goods supply chain design problems that are associated with uncertainties.

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