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The optimization model and empirical analysis for vehicle routing problems of s-company with time windows based on C-W algorithm

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Abstract: Vehicle routing problem is a kind of optimization scheduling problems which research how to realize transportation cost optimization though programming driving route reasonably, this paper through analyzing distribution system of S company to find the vehicle routing problems. Considering the vehicle routing problems with time window restriction, we combine the typical practical situation to establish the proper model. Based on c-w saving algorithm. The case study indicates that saving algorithm is reasonable, and it also showed that this calculation might be simple and convenient and this method could be easy to realize by computer. However, the precision of the solution will be reduced if the number of customers increases with increasing solution spaces.

Keywords: Vehicle Routing Problem, Time Windows Restriction, C-W Saving Algorithm, Logistics Distribution

1. Introduction

The Vehicle Routing Problems (VRP) was first proposed by Dantzig and Ramser in 1959. Vehicle Routing Problem with Time Window (VRPTW) is increased the customer access time window constrain on the basis of Vehicle Routing Problem, the constraint allows Vehicle Routing Problem with Time Window closer to the reality of logistics distribution than Vehicle Routing Problem. It is closer to the reality of logistics distribution to describe and deal with Vehicle Routing Problem with Time Window constrain than to do that with Vehicle Routing Problem. At the same time, the research for VRPTW has been paid more and more attention, Qu Qianqian solved Vehicle Routing Problem with Time Window by Hybrid Genetic Algorithm; Ho S c effectively solved the Vehicle Routing Problem with Time Window of 100 user by tabu search algorithm; Ma Huawei solved the Vehicle Routing Problem with Optional Time Windows by improved tabu search algorithm.

In general, on the one hand, it can improve the efficiency of logistics by solving the Vehicle Routing Problem with Time Windows(Li et al.,2010;Wang et al.,2011;Chen et al.,2005), on the other hand it can provide the transportation security for the enterprise of mode of production, and realizes the scientific management of logistics. The goal of Vehicle Routing Problem is achieving the minimum transportation cost and the minimum time, the problem can be transformed into the problem of multi-objective optimal operation of fully-loaded vehicle of multi-type vehicles multi- cargo kinds.

The paper firstly analyzed distribution system of S company of Henan Province, and then raises Vehicle Routing Problem with Time Windows that meets the constraints of the multi-objective, simultaneously, considering customer's time requirements, and establishes the proper model, and uses the model to solve the practical problem at last(Xiao,2008).

2. Analysis of Distribution System of S-company

2.1. Business Model of Distribution System of S-company

The Distribution system of S-company is self-support logistics. And all of S-company’s logistics and distribution business of is operated by itself. S-company is a chain enterprise, which controls the materials from the supplier, league members to the users. So enterprise logistics activities include two categories: supply logistics and distribution logistics. The company built distribution centers and warehouses, and purchased the distribution vehicles to complete the company's supply logistics and procurement. The distribution center is responsible for distribution system of S company.

2.2. The Research for Vehicle Routing Problem of S-company

1. Vehicle routing and distribution routing are not scientific

The choice of distribution route was mainly based on experience, subjective judgment. Distribution route from head office to branch was decided by their experience or habits of carriage driver. General practice to choose distribution route from branch to shops: at first, managers of distribution centers marked the shops on the map; then, partitioned into distribution sites to same route in accordance with the idea of ”neighboring”; lastly, the driver and the salesman actually had been to each distribution site, so the distribution route was adjusted by combining the route driver and the salesman to bring up, and it was used as the final distribution route. the lowest total distribution costs, the minimum total distribution time, the shortest total distribution mileage and other transportation scheduling and vehicle routing planning objectives for the distribution route can't be achieved.

2. High ratio of empty car kilometers to loaded car kilometers

There are three main reasons for high ratio of empty car kilometers to loaded car kilometers: firstly, Vehicle Routing is unreasonable. Distribution area is unreasonable, distribution route is irrelevancy, they can cause the high ratio of empty car kilometers to loaded car kilometers. Secondly, distribution system is autarky self-support logistics mode, it is to rely on their own vehicles for distribution and taking delivery, they can't find supply of goods for return. Thirdly, vehicle use is too special. Due to transport no cargo for return and the empty trucks drive in a single pass.

3. Route distribution volume and vehicle load don't match

The S-company uses means of distribution of appointed timing and route, every route is designated for a vehicle, goods amount that vehicle can load is fixed. But in practical applications, demand quantity of a number of distribution sites in the same route are different, so the distribution volume of the route are different, and sometimes the vehicle is empty, sometimes it can't load with goods. The problem is that every route is designated a specified fixed distribution vehicles, distribution volume of every route is a change, but load capacity of vehicle is fixed, so it leads to difficulties in operation.

3. Vehicle Routing Optimization Modeling

3.1. Model

In order to make the model simplification and to solve it conveniently, the following assumptions and explanation is essential: distribution center has some same type and load(Q) vehicles; it is assumed that each vehicle starts from the distribution center to deliver goods to the designated customer along the a specific route, and then returns to the distribution center; the goods that every customer needs can only be transported by a vehicle; different customers have different requirements on the time of arrival; it is assumed that all expense is related to transportation distance, and independent of other factors(Zhang et al.,2009).

When the model is built, first the distribution center is numbered as O, different distribution terminal are numbered as 1,2, ... , N, distribution center and distribution terminal all are numbered as point i(i=1,2…N).Parameter symbolic and variables of the model are defined as follows:

M: M vehicles， number : 1， 2， …， M； β： distribution costs of the average unit distance; g_i: distribution requirements of each distribution terminal *i*; Q: average load of vehicle; d_{ij}: the distance from i to j;c_2: Opportunity cost of unit time that causes by transportation vehicle waiting at the joint。If there is flexible time window constrain, its value is an identified a specific value; if there is hard time window constrain, its value is a large number, but in order to calculate conveniently, when we solve, we can take appropriate number in practical applications;c_3: value of unit time in which to transportation vehicles arrive the distribution terminal after the specified time. t_{ijk}: the time that the k vehicle takes from i to j, and ignoring the loading time of each point and traffic conditions and other factors, only related to transportation distance; x_{ijk}: decision variable, which indicates the k vehicle drive from i to j, if it is from i to j, the value of x_{ijk} is 1; or it is 0; y_{ijk} : decision variable, which indicates the task of distribution site is competed by the vehicle K, if it is K, the value of y_{ijk} is 1; or it is 0; the arrival time that customer requires can be general expressed as:$ET_i \le T_i \le LT_i$. Among them, ET_i is the starting point of the arrival time that customer requires, LT_i is the end point of the arrival time that customer requires, T_i is the time that the goods is transported by estimating.

The vehicle routing problem of mathematical modeling is as follows:

Objective function:

$$\mathrm{Min}\left\{\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=1}^{M}\beta d_{ij}x_{ijk} + \sum_{i=1}^{N}\mathrm{Max}[ET_i - T_i, 0] + c_3\sum_{i=1}^{N}\mathrm{Max}[T_i - LT_i, 0]\right\} \quad (3\text{-}1)$$

$$\begin{cases} \sum_{i=1}^{N} g_i y_{ik} \le Q, k = 1,2 \dots M & (3-2) \\ \sum_{k=1}^{M} y_{ik} = 1, i = 1,2 \dots N & (3-3) \\ \sum_{k=1}^{M} y_{0k} = M & (3-4) \\ \sum_{i=0}^{N} x_{ijk} = y_{ik}, j = 1,2 \dots N, k = 1,2 \dots M & (3-5) \\ \sum_{j=0}^{N} x_{ijk} = y_{ik}, i = 1,2 \dots N, k = 1,2 \dots M & (3-6) \\ T_i = \sum_{i=0}^{N} x_{ijk} t_{ijk} & (3-7) \\ x_{ijk}\left(T_j - T_i\right) \ge 0, i,j = 1,2 \dots N, k = 1,2 \dots M & (3-8) \end{cases}$$

A brief description of the above formula: Objective function(3-1) is the function for minimum cost, which the first term is total transportation cost, the second term is opportunity cost of waiting, if vehicle arrives early at distribution terminal, the third term is value, if vehicle arrives later than the time required at distribution terminal. constraints(3-2) expresses that the task load of vehicle k is less than or equal to load of the vehicle; constraints(3-3) expresses that task of each distribution terminal only be completed by a vehicle; constraints(3-4) expresses that M vehicles are sent from distribution center; constraints(3-5) and constraints(3-6) express the relationship between two variables; constraints(3-7) supplies mathematical expression of Ti in objective function; constraints(3-8) ensures succession of the distribution routes.

3.2. C-W Saving Algorithm Based on Time Window

1. Algorithm principle

Because the above model involves the NP-Hard, it is difficult to solve the problem, if we use accurate algorithm, we map out to use C-W saving algorithm of heuristic algorithm to solve the problem(Li & Guo,2009). Its basic idea is: firstly the distribution terminal is connected with distribution center, which constructed transport routes containing only a distribution terminal, and we calculate the transporting cost; then, we obtain the saving distance when any two distribution terminals are connected in a route, value represents total saving distance when two distribution terminals are connected in a route(Kennedy & Kopp,2011).

$$s(i,j) = c_{io}+c_{0j}-c_{ij} \qquad (3\text{-}9)$$

$c_{ij}(\beta d_{ij})$represents the cost from point i to j for each vehicle, by C-W saving algorithm, we obtain the cost saving value when point i and point j are connected together: $s(i,j) = c_{io}+c_{0j}-c_{ij}$

When we calculate it, we need to take into account two aspects: first, the vehicle capacity constraints, and freight volume of a route's each task is less than or equal to the vehicle capacity; time window constraints, if we need to complete the task in a certain period of time, when we connect the point i and j according to cost saving value, it cannot meet time requirements when the next tasks are performed, so we need consider the time window constrains. When we connect point i and j in a route, if the vehicle reaches the point j earlier than task starting of point j in the original route, the vehicle may wait before the next task are performed; if the vehicle reaches the point j later than task starting of point j in the original route, the vehicle may wait before the next task are performed; the next task can be delayed.

We use EF_j to represent the route that connecting point i and j is on, the delaying time that the vehicle reaches the point j later than point j in the original route can be obtained the following formula:

$$EF_j = T_i + t_{ij} - T_j \qquad (3\text{-}10)$$

Obviously, $EF_j<0$, vehicle reaches the point j early; $EF_j=0$, vehicle reaches the point j on time; $EF_j>0$, vehicle reaches the point j late.

In order to illustrate the problem, now the parameters are defined as follows:

$$\Delta_j^- = \min_{i\geq j}\{T_\gamma - ET_\gamma\} \quad (3\text{-}11)$$

$$\Delta_j^+ = \min_{i\geq j}\{LT_\gamma - T_\gamma\} \quad (3\text{-}12)$$

Δ_j^-—— maximum allowable amount of arriving time in advance that vehicle doesn't need to wait for point j when each task behind point j on the route is performed.

Δ_j^+—— maximum delaying amount of arriving time that each task behind the point j on the route does not violate time window constraint points.

Δ_j^- and Δ_j^+ can be calculated according to the following formula.

$$\Delta_j^- = \min_{i\geq j}\{T_\gamma - ET_\gamma\} \quad (3\text{-}11)$$

$$\Delta_j^+ = \min_{i\geq j}\{LT_\gamma - T_\gamma\} \quad (3\text{-}12)$$

When we examine the route that connect point i and j, it is necessary to check whether the transport time violates time window constraints.

When $EF_j<0$, if $EF_j \leq \Delta_j^-$, it doesn't need to wait when vehicle performs the task behind j, or it wili need to wait for some time;

When $EF_j>0$, if $EF_j \leq \Delta_j^+$, it doesn't delay when vehicle performs the task behind j, or it will delay.

2. Calculation and analysis

It is assumed that distribution centers chooses the model's result, 12 distribution tasks are assigned to the distribution center of Zhengzhou City, distribution demand of the distribution sites is (unit: ten thousand pieces). Because distribution center is located in Zhengzhou City, (denoted 0), level 2 distribution center is not established in Zhengzhou city, distribution center is responsible for distributing goods in Zhengzhou. The tasks are performed by the vehicles sent from distribution center what a total capacity Q is 30000 pieces, characteristics of distribution sites and requirements in table 1.

Table 1: Characteristics and requirements of tasks on different distribution sites

Distribution site i	0	1	2	3	4	5
city	Zhenzhou	Luoyang	Kaifeng	Xuchang	Zhoukou	Luohe
g(ten thousand pieces)	0.5	1.0	1.4	0.8	0.5	0.3
[ET,LT]	[5,10]	[5,15]	[7,17]	[6,12]	[5,15]	[9,17]
Distribution site i	6	7	8	9	10	11
city	Ping ding shan	Xin xiang	Shang qiu	Zhu ma dian	Xin yang	San men xiang
g(ten thousand pieces)	1.2	1.5	0.2	0.6	0.3	0.5
[ET,LT]	[12,27]	[10,20]	[7,18]	[9,18]	[8,20]	[10,20]

It is assumed that travel time of vehicle is proportional to distance, speed of the vehicle is set to 50 km / h, the travel time from i to j is $t_{ij}=d_{ij}/50$. Initially, when the vehicle drives from the distribution center in Zhengzhou city to distribution sites, if $ET_i \le t_{0i} \le LT_i$, and $T_i=t_{0i}$,if $t_{0i} \le ET_i$,and $T_i=ET_i$, then we calculate it according to C-W saving algorithm, and make frame table of route, we intercept part of the calculation process, as shown in Table 2 follows:

Table 2: Lines structure process table

i-j	position of two points	$q=\sum g_i$	$EF_{ij}=T_i+t_{ij}-T_j$	Δ_i^- or Δ_i^+	Connection type	$T_k=T_k+EF_j$ ($k\ge j$)
8-11	Point that isn't on the line	q=0.7<Q	EF_{11}=-0.9	$\Delta_{11}^-=1$	8→11	T_{11}=10.1
11-10	Point that isn't on the line Lateral	q=1<Q	EF_{10}=5.62	$\Delta_{10}^+=$ 10.4	8→11→10	T_{10}=15.2
11-4	A point is inner point				No	
10-8	On the same line				No	
10-7	Point that isn't on the line Lateral	q=2.5<Q	EF_7=14.1	$\Delta_7^+=10$	No	
9-8	Point that isn't on the line	q=1.6<Q	EF_8=9.4	$\Delta_8^+=4.78$	No	
10-9	Point that isn't on the line Lateral	q=1.6<Q	EF_9=16.38	$\Delta_9^+=9$	No	

There are four lines by Calculation:

0 → 4 → 2 → 3 → 5 → 0 923km

0 → 1 → 0 60km

0 → 7 → 9 → 0 1149km

0 → 8 → 11 → 10 → 6 → 0 1500km

We get the total distribution mileage is 3632 km by calculating , it can save nearly half of the mileage compared to sent goods for each user individually (total 6520 km),as show in figure 1.

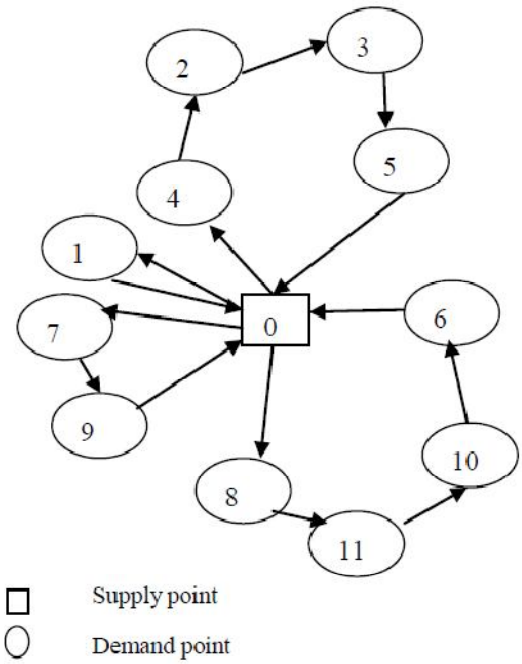

Fig. 1: General drawing of distribution route planning

4. Conclusions

It is shown from the analysis, the premise that the distribution tasks are completed successfully is that the lowest cost of target distribution, the distribution cost and distribution services reach a balance. But in the algorithm there is some disadvantages that some edge points are difficult for combining to affect optimization rate, it is also necessary in the future to determine the research to modify the method, in order to make it suitable for solving with Vehicle Routing Problem of under capacity logistics distribution vehicle with time window constraints.

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