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An integrated optimization model of a Closed-Loop Supply Chain under uncertainty

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Abstract: This paper studies a closed-loop supply chain, mainly consisting of positive selling, repairing, and remanufacturing under uncertainty. Then, we build a model for the closed-loop supply chain integration and optimization under uncertainty, in which a comprehensive utility function is used to clarify the random parameters, so that we can transform the model into a mixed integer programming model and do further analysis by hybrid genetic algorithm. The feasibility and validity of the model are verified by illustrative examples. This paper provides an effective decision-making tool for the closed-loop supply chain design. It is useful for the integration of forward and reverse supply chains.

Keywords: Closed-loop supply chain, Products repair, Modular reutilization, Uncertainty environment, Hybrid genetic algorithm

1. Introduction

Early supply chains were mostly profit-oriented and aimed at reducing costs and enhancing competitiveness. They had one-way linear structure which lacked the awareness of sustainable development. However, through the growth of the global market, more and more companies realized that customers' satisfaction is extremely important for them to survive. The development of the buyer's market leaded to fiercer competition by increasing product alternatives and shortening products life cycles. The more and more important role of the market, especially the role of consumers' choice, has forced companies to extend products liability to a longer period, sometimes even lifelong. Therefore, the supply chain requires innovation urgently. A series of operating procedures and associated networks should be added to the traditional supply chains, such as testing, repairing, reprocessing, and recycling. A new supply chain will thus be constructed by integrating the traditional one with various reverse activities. In this new chain, all materials of the products in the circular flow should be managed effectively in their life cycles, thus the negative impacts on environment would be reduced. Such a supply chain is named as Closed-Loop Supply Chain (CLSC) (Harold, K., et al., 2001, Van Wassenhove, L. N., et al., 2003). The closed-loop supply chain consists of both the positive supply chain and the reverse supply chain in the product life cycle. There are a number of methods that we can use to construct supply chains.

The closed-loop supply chain networks are mostly under uncertainties. Most studies established optimized model of closed-loop supply chain in a supposed approximation of certainty; but it is against the existence of the uncertain information such as vagueness or randomness, such as Mao H.J., et al., (2006); Liu, Q., et al., (2007); El-Sayed, et al.,(2008), and so on. In this paper, by taking the random environmental factors into consideration, we will establish the model, which it is based on the uncertainties of products repair, recycling, sales, and the time value of money.

Afterwards, we will give numerical examples to present the effects of parameters on the closed-loop supply chain model under uncertainty, the results indicate that the model is not only accommodates the existing uncertainty decision-making methods, but also successfully incorporates the decision preference into the optimization process.

2. Modeling design

2.1. Decision variables and parameters description

Consider varieties of products and the fact of modular production. Every factory can make and repair goods at the same time, but the unit costs of production and repairing are different; during every period, demands of consumer markets for various products are treated as random numbers. Demands among consumer markets are independent; the demand of consumers for each product can be supplied partially or excessively. Both situations will induce penalty cost; the number of tested products and recycled products is proportional to the demands of consumers. Only useful products will be recycled; all the facilities will be built at known locations, and restrictions are set on capacity constraints; all facilities should be constructed or extended in the corresponding periods, according to the demands of consumer markets. But the extension scale is limited, and the time spent is negligible; for each facility, the handling costs, transportation costs and penalty costs are known; selected facilities cannot be removed but can only be extended.

In this section, symbols for parameter variables and decision variables in the model are as follows: I Number of factories locations,$i\epsilon\{1,2,\cdots,I\}$, J Number of disassembling centers locations; $j\epsilon\{1,2,\cdots,J\}$ M Number of customers locations, $m\epsilon\{1,2,\cdots,M\}$; N Number of testing center locations, $n\epsilon\{1,2,\cdots,N\}$; T Number of time periods, $t\epsilon\{1,2,\cdots,T\}$; L Number of product varieties, $l\epsilon\{1,2,\cdots,L\}$; Q_{ablt}^{cd} Number of product l that transports from a to b during the t period ; X_{at}^{c} If a new facility c is built in location a during the t period, then $X_{at}^{c}=0$; Y_{at}^{c} If the facility c is extended in location a during the t period, then $Y_{at}^{c}=1$; otherwise $Y_{at}^{c}=0$; Z_{at}^{c} The scale of the facility c extended in location a during the t^th period; FC_{at}^{c} The cost of building a new facility c in location a during the t^th period; KC_{at}^{c} The cost of extending one unit scale of facility c in location a during the t^th period; VC_{ilt}^{pm} the cost of producing one unit of product l in factory i during the t^th period; VC_{ilt}^{pr} The cost of repairing one unit of product l in factory i during the t^th period; VC_{alt}^{c} The cost of disposing one unit of product l by facility c in location a during the t^th period; SC_{ablt}^{cd} The cost of transporting one unit of product l from a to b during the t^th period; A_{a}^{c} The maximal capacity of facility c in location; B_{a}^{c} The increased capacity of extending one unit of scale of facility c in location; C_{a}^{c} The maximal scale of extending facility c in location; D^{c}The maximal number of new facility c that can be built; U_{il}^{pm} The capacity cost of producing one unit of product l in factory i; U_{il}^{pr} The capacity cost of repairing one unit of product l in factory i; U_{al}^{c} The capacity cost of disposing one unit of product l by facility c in location a; D_{mlt} The demand of custom market m' for product l during the t^th; λ_{mlt}^{+} The penalty cost one unit of product l when the supply of it is greater than the demand of custom market m during the t^th period; λ_{mlt}^{-} The penalty cost one unit of product l when the supply of it is less than the demand of custom market m during the t^th period; r_{mlt} The failure (return) rate of product l in custrom market m during the t^th period; g_{mlt} The recycling rate of product l in custom market m during the t^th period; e_{mlt} The failure (return) rate of product l which need to be repaired in custom market m during the t^th period; θ_{mlt} The average value of one unit of product l recycled in custom market m during the t^th period; IR_t The Annual Percentage Rate (APR) of the bank during the t^th period. The subscripts a and b in decision variables and parameters; *i*, *j*, *m* and *n* symbolize the location of the new built factory, the new built disassembling center; the consumer market location and the testing center location, respectively; The superscript c and d in decision variables and parameters; *p*, *w*, *z* and *r* symbolize factories, disassembling center, consumer markets and testing centers respectively.

2.2. Network optimization model

The objective functions of the model:

$$\min TC = FixC + OperC + TranC + PuniC - ModC \qquad (1)$$

$$FixC = \sum_{t=1}^{T} [(\sum_{i=1}^{I} FC_{it}^{p} X_{it}^{p} + \sum_{j=1}^{J} FC_{jt}^{w} X_{jt}^{w} + \sum_{n=1}^{N} FC_{nt}^{r} X_{nt}^{r}) \times (1+IR_t)^{-(t-1)}]$$
$$+ \sum_{t=1}^{T} [(\sum_{i=1}^{I} KC_{it}^{p} Z_{it}^{p} Y_{it}^{p} + \sum_{j=1}^{J} KC_{jt}^{w} Z_{jt}^{w} Y_{jt}^{w} + \sum_{n=1}^{N} KC_{nt}^{r} Z_{nt}^{r} Y_{nt}^{r}) \times (1+IR_t)^{-(t-1)}] \qquad (2)$$

$$OperC = \sum_{t=1}^{T} \{[\sum_{l=1}^{L} \sum_{i=1}^{I} VC_{ilt}^{pm} \sum_{m=1}^{M} Q_{imlt}^{pz} + \sum_{l=1}^{L} \sum_{i=1}^{I} VC_{ilt}^{pr} \sum_{n=1}^{N} Q_{nilt}^{rp}$$
$$+ \sum_{l=1}^{L} \sum_{j=1}^{J} VC_{jlt}^{w} \sum_{m=1}^{M} Q_{mjlt}^{zw} + \sum_{l=1}^{L} \sum_{n=1}^{N} VC_{nlt}^{r} \sum_{m=1}^{M} Q_{mnlt}^{zr}] \times (1+IR_t)^{-(t-1)}\} \qquad (3)$$

$$TranC = \sum_{t=1}^{T}[(\sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{i=1}^{I} SC_{imlt}^{pz}Q_{imlt}^{pz} + 2\sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{n=1}^{N} SC_{nilt}^{rp}Q_{nilt}^{rp} + \sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{m=1}^{M} SC_{mjlt}^{zw}Q_{mjlt}^{zw} + \sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{j=1}^{J} SC_{jilt}^{wp}Q_{jilt}^{wp}) \times (1+IR_t)^{-(t-1)}] \quad (4)$$

$$PuniC = \sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{m=1}^{M} \max\left\{\sum_{i=1}^{I} Q_{imlt}^{pz} - E(D_{mlt}), 0\right\} \times \lambda_{mlt}^{+} + \sum_{t=1}^{T}\sum_{l=1}^{L}\sum_{m=1}^{M} \max\left\{E(D_{mlt}) - \sum_{i=1}^{I} Q_{imlt}^{pz}, 0\right\} \times \lambda_{ml}^{-} \quad (5)$$

$$ModC = \sum_{t=1}^{r}[(\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{m=1}^{M} Q_{mjlt}^{zw}\theta_{mlt}) \times (1+IR_t)^{-(t-1)}] \quad (6)$$

Formula (2) expresses the fixed cost of building and extending factories, disassembling centres and testing centres. Formula (3) expresses the operating cost of all the facilities. Formula (4) expresses the transportation cost among all the facilities Formula (5) expresses the penalty cost when the supply is either greater or less than the demand. Formula (6) expresses the re-use value of the recycled products.

Constraints:

$$\sum_{l=1}^{L}(\sum_{m=1}^{M} Q_{imlt}^{pz}U_{il}^{pm} + \sum_{n=1}^{N} Q_{nilt}^{rp}U_{il}^{pr}) \le \sum_{\theta=1}^{t} A_i^p X_{i\theta}^p + \sum_{\theta=2}^{t} B_i^p Z_{i\theta}^p Y_{i\theta}^p \quad \forall i,t \quad (7)$$

$$\sum_{l=1}^{L}(\sum_{m=1}^{M} Q_{mjlt}^{zw}U_{jl}^{w}) \le \sum_{\theta=1}^{t} A_j^w X_{j\theta}^w + \sum_{\theta=2}^{t} B_j^w Z_{j\theta}^w Y_{j\theta}^w \quad \forall j,t \quad (8)$$

$$\sum_{l=1}^{L}[(\sum_{m=1}^{M} Q_{mnlt}^{zr} + \sum_{i=1}^{I} Q_{inlt}^{pr})U_{nl}^{r}] \le \sum_{\theta=1}^{t} A_n^r X_{n\theta}^r + \sum_{\theta=2}^{t} B_n^r Z_{n\theta}^r Y_{n\theta}^r, \forall n,t \qquad Q_{mnlt}^{zr} = Q_{nmlt}^{rz}, \forall n,m,l,t$$

$$\sum_{i=1}^{I} Q_{jilt}^{wp} \le \sum_{m=1}^{M} Q_{mjlt}^{zw} \quad \forall j,l,t, \quad \sum_{i=1}^{I} Q_{inlt}^{pr} \le \sum_{m=1}^{M} Q_{nmlt}^{rz}, \forall n,l,t, \quad \sum_{i=1}^{I} Q_{nilt}^{rp} = \sum_{i=1}^{I} Q_{inlt}^{pr}, \forall n,l,t \quad (9)$$

$$\sum_{i=1}^{I} Q_{nilt}^{rp} = \sum_{m=1}^{M} e_{mlt} Q_{mnlt}^{zr}, \forall n,l,t \tag{10}$$

$$\sum_{n=1}^{N} Q_{nilt}^{rp} \le M \sum_{\theta=1}^{t} X_{i\theta}^{p}, \forall i,l,t, \sum_{j=1}^{J} Q_{jilt}^{wp} \le M \sum_{\theta=1}^{t} X_{i\theta}^{p}, \forall i,l,t, \sum_{m=1}^{M} Q_{imlt}^{pz} \le M \sum_{\theta=1}^{t} X_{i\theta}^{p}, \forall i,l,t$$

$$\sum_{m=1}^{M} Q_{mjlt}^{zw} \le M \sum_{\theta=1}^{t} X_{j\theta}^{w}, \forall j,l,t, \sum_{m=1}^{M} Q_{mnlt}^{zr} \le M \sum_{\theta=1}^{t} X_{n\theta}^{r}, \forall n,l,t, \sum_{m=1}^{M} Q_{nmlt}^{rz} \le M \sum_{\theta=1}^{t} X_{n\theta}^{r}, \forall n,l,t$$

$$\sum_{i=1}^{I} Q_{nilt}^{rp} \le M \sum_{\theta=1}^{t} X_{n\theta}^{r}, \forall n,l,t \tag{11}$$

$$\sum_{n=1}^{N} Q_{mnlt}^{zr} = r_{mlt} E(D_{mlt}), \forall m,l,t \tag{12}$$

$$\sum_{j=1}^{J} Q_{mjlt}^{zw} = g_{mlt} E(D_{mlt}), \forall m,l,t \tag{13}$$

$$\sum_{t=1}^{T} X_{it}^{p} \le 1, \sum_{t=1}^{T} X_{jt}^{w} \le 1, \sum_{t=1}^{T} X_{nt}^{r} \le 1, \forall i,j,n \tag{14}$$

$$\sum_{t=1}^{T} \sum_{i=1}^{I} X_{it}^{p} \le D^{p}, \sum_{t=1}^{T} \sum_{j=1}^{J} X_{jt}^{w} \le D^{w}, \sum_{t=1}^{T} \sum_{n=1}^{N} X_{nt}^{r} \le D^{r} \tag{15}$$

$$Y_{it}^{p} \le M \sum_{\theta=1}^{t-1} X_{i\theta}^{p}, \quad Y_{jt}^{w} \le M \sum_{\theta=1}^{t-1} X_{j\theta}^{w}, \quad Y_{nt}^{r} \le M \sum_{\theta=1}^{t-1} X_{n\theta}^{r}, \forall i,j,n,t \in \{2,\ldots,T\} \tag{16}$$

$$Z_{it}^{p} \le M Y_{it}^{p}, \quad Z_{jt}^{w} \le M Y_{jt}^{w}, \quad Z_{nt}^{r} \le M Y_{nt}^{r} \quad, \forall i,j,n,t \in \{2,\ldots,T\} \tag{17}$$

$$\sum_{t=2}^{T} Z_{it}^{p} \le C_{i}^{p}, \quad \sum_{t=2}^{T} Z_{jt}^{w} \le C_{j}^{w}, \quad \sum_{t=2}^{T} Z_{nt}^{r} \le C_{n}^{r} \quad, \forall i,j,n \tag{18}$$

$$Q_{ijlt}^{pw}, \ Q_{inlt}^{pr}, \ Q_{imlt}^{pz}, \ Q_{nilt}^{rp}, \ Q_{jilt}^{wp}, \ Q_{nmlt}^{rz}, \ Q_{mnlt}^{zr}, \ Q_{jmlt}^{wz}, \ Q_{mjlt}^{zw} \ge 0 \quad \forall i,j,m,n,l,t \tag{19}$$

$$X_{it}^{p},\ X_{jt}^{w},\ X_{nt}^{r} \in \{0,1\}\ \forall i,j,n,t\quad Y_{it}^{p},\ Y_{jt}^{w},\ Y_{nt}^{r} \in \{0,1\}\ \forall i,j,n,t \in \{2,\ldots,T\}$$

$$Z_{it}^{p},\ Z_{jt}^{w},\ Z_{nt}^{r} \geq 0, \forall i,j,n,t \in \{2,\ldots,T\} \qquad (20)$$

Constraints (7) and (8) describe the limited capacity of each facility; Constraints (9) assure the amount of input and output logistics should conserve. Constraint (10) presents the number of product need to be repaired. Constraints (11) assure that a facility has input and output logistics only if the facility has already been set up. M is an infinite number. Constraint (12) assures the total number of products customer returns is equal to the number of market expected value of repairing. Constraint (13) assures the total number of products disassembled is equal to the number of market expected value of recovery. Constraint (14) presents that each facility is only built once. Constraint (15) describes that the number of facilities is limited. Constraint (16) assures that each facility would be extended only if it already exits. Constraint (17) assures the extending scale of the facilities is a positive integer only if they are to be extended. Constraint (18) assures the extending scale cannot exceed the maximal scale. Constraints (19) to (20) present the ranges of each decision variables.

3. Numerical results

An equipment enterprise considers building a closed -loop supply chain network which will include both distribution channels and the reverse logistics network of product returns and recycling. We consider three periods, three candidates of factories, five candidates of testing centers, two candidates of dismantling centers, and 10 consumer markets in our design. Parameters of customer demands are assumed to have normal distribution the data of other Parameters is omission. In order to solve the model, we design and prepare the optimized program with the genetic algorithm optimal-toolbox of MATLAB, Parameters of the designed GA are: rnax_generation 100, poi_size 80, cross_rate 0.5, mutation_rate 0.1, and the generation gap 90%. In this paper, the random data is clarified by the third random effects comprehensive function.

Then, we use this algorithm to solve the above-mentioned model. Results after calculation are shown in the Table 1.

Table 1The summary of the solution in the model

	Select F			Select TC					Select DC		extend F			extend TC					extend DC	
t=1	0	1	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0
t=2	0	1	0	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0	2	0
t=3	0	1	0	0	1	1	1	1	1	0	0	1	0	0	0	1	0	0	0	0

The total cost of SCLSC model is $1.6992E +06, in which the fixed capital cost is $1.6879E +06, the extension cost is $1.0376E +04, penalty cost is $783.00, transportation cost is $3.2731E +03; the utilizing value generated by the recovery and demolition is $735.14. In short, in the results from multi-period optimization model, facilities are relatively small and centralized. Therefore, it is good for the operation and management of user-friendly closed-loop supply chain.

4. Conclusions and future work

This paper mainly focuses on the optimized design of closed-loop supply chain in uncertain environment. It offers dynamic model of location selection based on different factors. It takes the dynamic state of location, facility extension, and capacity improvement into consideration. The feasibility and efficiency has been analyzed and verified through cases. Numerical results show that the algorithm based on optimal perturbation analysis is feasible and effective. We also analyzed some parameters of the model.

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