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Influence of actors in alliance game based on social network analysis theory and its application

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Abstract: Shapley value reflects the influence of the actors in alliance game. Traditional method of calculating the Shapley value takes the actors' absolute power only. In this paper, Social Network Analysis (SNA) method is introduced to concern about the interactive function. The traditional Shapley value is modified by relatively point centrality. An example given at the end of the paper shoes the process of the method.

Keywords: Social Network Analysis (SNA), Relatively Point Centrality, Alliance Game, Shapley Value

1. Introduction

Shapley is one of the founders of the game theory. In 1953 he studies the problem of non-strategy multi-person cooperative game theory (Shapley, 1953). Shapley value is a well-known solution concept in this problem. Suppose a situation where if some economic agents make up a cooperative relationship, i.e., a coalition, then they can get more gains than those if they do not do so. In such situations, one of people's interests is how much share each of them should get by forming the coalition. Shapley value shows a vector whose elements are agents' share derived from some reasonable bases (Shapley, 1953;Aumann and Shapley,1974; Roth, 1988).

Shapley value has been investigated by a number of researchers. There is a research presents a new approximation algorithm, based on randomization, for computing the Shapley value of voting games. It also evaluates the error for the method and shows how the different parameters of the voting game affect it (Fatima, Wooldridge and Jennings, 2008). Liben-Nowell ,D., Sharp,A., Wexler, T. and Woods, K consider the class of supermodular games, and give a fully polynomial-time randomized approximation scheme (FPRAS) to compute the Shapley value to within a $(1 \pm \varepsilon)$ factor in monotone supermodular games. They show that this result is tight in several senses: no deterministic algorithm can approximate Shapley value as well, no randomized algorithm can do better, and both monotonicity and supermodularity are required for the existence of an efficient $(1 \pm \varepsilon)$ approximation algorithm(Liben-Nowel, Sharp, Wexler and Woods, 2012). Chun, Y. and Park, B. present a new axiomatic characterization of the Shapley value on the basis of strong coalitional fair-ranking: it requires that if a game is added to another game with two symmetric players, then the relative positions of the symmetric players should not be affected. The Shapley value is the only value satisfying strong coalitional fair-ranking together with efficiency, the null player property and strategic equivalence(Chun & Park, 2011).And there is a research that using the Shapley Value to price multi-class network services(Zhang & Verma,2011). Guiasu, S tries to show that the Shapley value from game theory, measuring the power of each player in a game, may be consistently applied for getting the general one-step solution of all these three problems (bankruptcy, contested garment, and rights arbitration) viewed as n-person games (Guiasu,2011). Meng, F. and Liu, F. research the interval shapley value for type-2 interval games (Meng & Liu,2012) . And José María Alonso-Meijide and Francesc Carreras research the proportional coalitional Shapley value (Alonso-Meijide and Carreras,2011).In addition, based on Shapley value, Owen adopted another approach characterizing axiomatically a coalitional value called now Owen value (Owen, 1977; Owen, 1995). In this case, the unions play a quotient game among themselves, and each one receives a payoff which, in turn, is shared among its players in an internal game. Both payoffs, in the quotient game for unions and within each union for its players, are given by the Shapley value. In addition to the initial one, many other axiomatic characterizations of the Owen value can be found in the literature (Albizuri,2008; Amer,1995; Amer,2001; Hamiache,1999; Hart and Kurz,1983; Vázquez-Brage,1997;Winter, 1992).

Shapley value reflects the players' power in the alliance. It is a solution of the classical cooperative game theory. Classical cooperative game based on two assumptions. One suggests that players participate some certain union completely, i.e., each player either join in a union or not. It does not be other probability for players participating or not-participating in an alliance. The other suggests players in the game be completely independent, without exchange of information, matter and energy. So the players make the decision all by themselves.

Actually, it is impossible for players being completely independent. Exchange of information, material and energy is unavoidable, which constitute a network. Considering the problem in the context of social network, seem more accurate to describe. This paper would introduce the method of Social Network Analysis (SNA), to modify Shapley value.

The organization is as follows. In second section, the framework of the paper is stated and a minimum of preliminaries is provided. In third section, we introduce SNA and modify the traditional Shapley value. A simply case is studied in fourth section. Finally, the fifth section collects some conclusions and describes prospect of the research.

2. Preliminaries

In this paper, we consider cooperative games with the set of players $N = \{1,2,..,n\}$. A coalition T is a nonempty subset of N, which is identified with a function v from N to $\{0,1\}$. Function v standing a contribution function, $v(T)$ means the payoff level of coalition T. For any coalition S, if $v(S \cap T) = v(S)$ satisfied, T is called the carrier of the game. Let π be an arrangement of N, define a game (N,π,υ), for any coalition $S = \{i_1, i_2, ..., i_s\}$, $U(\pi(i_1), \pi(i_2), ..., \pi(i_s)) = v(S)$ satisfied.

Theorem 1 (Optimum or Validity) Let $\Phi_i(T)$ denotes the payoff sum of all players.

If T is the carrier of v, then $\sum_{i=1} \Phi_i(v) = \Phi_i(T)$;

Theorem 2 (Symmetry) If two players substitute each other, the payoff is unalterable, i.e. for any π of N, $\Phi_{\pi_i}(\pi v) = \Phi_i(v)$ satisfied.

Theorem 3 (Additivity) The payoff of the sum of two games equals to the sum of the payoff of two games. i.e. For u ,$v \in G$ $\Phi_i(u+v) = \Phi_i(u) + \Phi_i(v)$ satisfies.

It could be proved that Shapley value is unique theoretically when the three theorems are satisfied.

$$\Phi_i(v) = \sum_{i \in S \subseteq N} \frac{(|S|-1)!(n-|S|)!}{n!}[v(S) - v(S \setminus \{i\})] \tag{1}$$

Where $|S|$ denotes the number of the players involved in the game, $S\backslash\{i\}$ denotes the set S without the player i.

3. Social Net and Modified Shapley Value

In a general opinion, it was Brown, the English anthropologist, who firstly put forward the conception of 'social net', which is the set made up of many nodes and segments between the nodes(Wasserman & Faust,1994;Lei & Xin,2011). The node can be a person, an organization, a team and even a country. In that, the SNA theory can be used to study different unit.

Social network analysis method uses absolute centrality and relative centrality to measure the individual status and influence in the network. Absolute centrality A_i refers to the numbers of connecting segments of individual i to others. More times individual connect to others, greater effect it would show and bigger the absolute centrality becomes. The absolute centrality expression is as follows

$$A_i = \frac{o_i + i_i}{2} \tag{2}$$

where o_i stands for out degree and i_i for in degree.

When we analysis individual position in network and its behavior tendency, absolute center of pure research are not of much significance. In this paper Shapley value is studied based on the relative centrality, which is normalized results of absolute centrality, and the expression is as shown in (3).

$$R_i = \frac{A_i}{\sum_i A_i} \tag{3}$$

In an alliance game, Shapley value and centrality in the network determined individual influence. So Shapley value can be modified with relative centrality.

$$\Gamma_i^{'} = \Phi_i(v) \cdot R_i = \\ \{\sum_{i \in S \subseteq N} \frac{(|S|-1)!(n-|S|)!}{n!}[v(S) - v(S \setminus \{i\})]\} \cdot \frac{A_i}{\sum_i A_i} \tag{4}$$

After normalizing (4), we can get (5).

$$\Gamma_i = \frac{\Gamma_i^{'}}{\sum_i \Gamma_i^{'}} \tag{5}$$

And we get the result Γ_i that has been modified. Connected with the status in networks of individual *i*, it would be able to show the real influence of individual.

The following steps show the process of the method.

Step 1：Calculate the Shapley value of individual *i* by (1);

Step 2：Calculate the absolute centrality and relative centrality of individual *i* by (2), (3);

Step 3：Calculate the modified Shapley value by (4), (5).

4. Case Studies

Considered a family of three generations includes seven people: grandpa, grandma, grandpa in law, grandma in law, father, mother and daughter. Usually when they have some decisions to be made, they would vote though negotiation. Generally subject can get through if more than four persons agree. However, being spoiled by adults, the daughter's suggestion would get through if anyone of the members agrees. Let's sign the grandpa, grandma, grandpa in law, grandma in law, father, mother and daughter with 1,2,3,4,5,6,7 corresponding. Let's note $N = \{1,2,3,4,5,6,7\}$. And we will get:

$v(S) = 1, \forall S \subseteq N, |S| \geq 4$,

$v(1,7) = 1$, $v(2,7) = 1$, $v(3,7) = 1$, $v(4,7) = 1$, $v(5,7) = 1$, $v(6,7) = 1$,

$v(1,2,7) = 1$, $v(1,3,7) = 1$, $v(1,4,7) = 1$, $v(1,5,7) = 1$, $v(1,6,7) = 1$,

$v(2,3,7) = 1$, $v(2,4,7) = 1$, $v(2,5,7) = 1$, $v(2,6,7) = 1$, $v(3,4,7) = 1$,

$v(3,5,7) = 1$, $v(3,6,7) = 1$, $v(4,5,7) = 1$, $v(4,6,7) = 1$, $v(5,6,7) = 1$

Under other conditions, we get $v(S) = 0$.

All family members influence could be quantitative analyzed by Shapley index. For example, for individual *i*, we should calculate the following subsets that contains 1: {1,2,3,4}, {1,2,3,5}, {1,2,3,6}, {1,2,4,5}, {1,2,4,6}, {1,2,5,6}, {1,3,4,5}, {1,3,4,6}, {1,3,5,6}, {1,4,5,6}, {1,7}.

$$\Phi_1(v) = 10 \times \frac{(4-1)!(7-4)!}{7!} + \frac{(2-1)!(7-2)!}{7!} = \frac{2}{21}$$

And in the same way we can get:

$$\Phi_2(v) = \Phi_3(v) = \Phi_4(v) = \Phi_5(v) = \Phi_6(v) = \frac{2}{21}, \Phi_7(v) = \frac{9}{21}.$$

So Shapley value of the family members is described as $(\frac{2}{21},\frac{2}{21},\frac{2}{21},\frac{2}{21},\frac{2}{21},\frac{2}{21},\frac{9}{21})$.

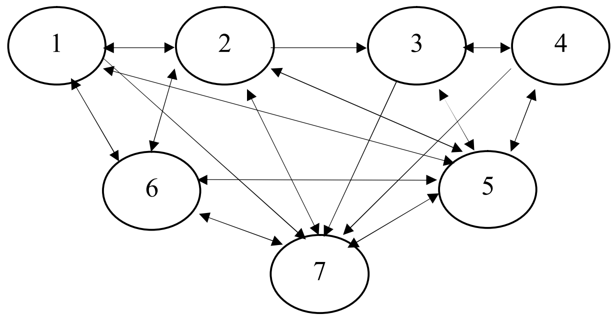

Fig. 1: Relations between the family members

The calculation process shown above does not take the relationship in networks into account. After observing the relations between the family members, the figure of relation networks can be drawn as Fig. 1.

According to (2), (3), we can calculate the centrality of the family members. The result of absolute centrality is shown as following:

$$A_1=\frac{7}{2},\ A_2=\frac{9}{2},A_3=\frac{6}{2},A_4=\frac{5}{2},A_5=\frac{12}{2},A_6=\frac{8}{2},A_7=\frac{9}{2},$$

And relative centrality as

$$R_1=0.125,R_2=0.161,R_3=0.107,R_4=0.089,R_5=0.214,R_6=0.143,$$
$$R_7=0.161$$

According to equation (4) and (5), modified Shapley value can be calculated and described as:

$$\Gamma_1=0.080,\Gamma_2=0.103,\Gamma_3=0.068,\Gamma_4=0.057,\Gamma_5=0.137,\Gamma_6=0.091,$$
$$\Gamma_7=0.462$$

By comparison with the results, we can see centrality has obvious effect on the power of members. Before modified by centrality, the Shapley values of grandpa, grandma, grandpa in law, grandma in law, father and mother equal to each other. On the contrary, the modified Shapley values are quite different. The tense relationship between parents-in-law and daughter-in-law (there is no information communication between node 3 and node 6, node 4 and node 6 as shown in Fig. 1) greatly weakened the grandparents' influence. Father has the capability to deal with both parties (node 5 has a two-way communication to any other node as shown in Fig. 1). So he has a bigger influence than the sum of that of grandma and grandpa. Granddaughter is the core of the family and she has more influence than that before modified.

5. Summaries

Study on the influence of individual in alliance game, should be in the framework of SNA. It is a helpful attempt to modify the Shapley value by relative centrality in this paper. The thought of the method is clear, simply to apply. From the case, we can see that modified results accord with reality well. From the perspective, this method can also be used in the enterprise alliance, integration of production, education and research and other related fields.

It is the first time to combine the game theory and social network analysis method, when the author study individual's network status considered the centrality only. Therefore the method is not perfect. More thorough discuss would be completed in future research.

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