

Absolute stability decision models of supplier selection problem under uncertain demand

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Abstract: Supplier selection is one key problem of logistics and supply chain management. Uncertainty of demand is the main challenges to the supplier selection. In this paper the stability of supplier selection was considered. Firstly the optimization model of supplier selection was given, then the absolute stability decision model of supplier selection was put forward, and finally an example was given.

Keywords: Supplier Selection Problem, Stability Decision, Robust Optimization, Optimal Model.

1. Introduction

Supplier selection is one key problem of logistics and supply chain management. The main research methods of supplier selection problem are comprehensive evaluation, cost analysis, mathematical programming (Lin & Ma, 2000; Liu, et.al., 2004; Chen & Qi, 2009).

The uncertainty of demand is the main challenges of supplier selection decision, general processing method is based on the average demand forecast. But it is difficult to give a precise prediction, the longer the time the greater of the prediction error. Sometime it can only give the range of demand, so need to study the supplier selection problem with the uncertainty demand.

The supplier selection problems with fuzzy demand and stochastic demand have been studied (Wang & Yang, 2009; Awasthi, et al., 2009; Li, et al., 2009). Fuzzy demand and stochastic demand need to know the probability distribution

function or the fuzzy membership function. But many cases it is difficult to get them. Thus supplier selection problem with completely uncertain demand should be considered.

The range programming (Guo, Jiang, & Qi, 2011) and robust optimization method (Ben-Tal & Nemirovski, 2002) are main methods to solve completely uncertain decision-making problem. These methods required decision to meet all the parameters of the possible changes in the constraints, or make a choice based on regret value of worst-case. To extreme cases the decision-making optimality will be relatively poor. We have given stability decision (Ma, 2011) to overcome the shortcomings. In this paper we will solve supplier selection problem under completely uncertain demand by stability decision method.

2. The supplier selection problem

Consider one enterprise selects raw material suppliers from m candidate enterprises. Each enterprise's maximum production capacity is $s_i^+, i=1,2,\dots,m$, the minimum contract order quantity is $s_i^-, i=1,2,\dots,m$, the unit transportation cost is $c_i, i=1,2,\dots,m$, and demand of the enterprise is d .

The variable $x_i, i=1,2,\dots,m$ indicates whether the i^{th} candidate enterprise is selected, $x_i = 1$ if it is selected, otherwise $x_i = 0$. The variable $y_i, i=1,2,\dots,m$ is contract order quantity of the i^{th} candidate enterprise. If $x_i = 0$ then $y_i = 0, i=1,2,\dots,m$. If the demand is certain, there are constraints:

$$\begin{aligned} s_i^- x_i \leq y_i \leq s_i^+ x_i, i=1,2,\dots,m \\ \sum_{i=1}^m y_i = d \end{aligned} \quad (1)$$

The total transportation cost is $\sum_{i=1}^m c_i y_i$, its programming model is:

$$\begin{aligned} \min \sum_{i=1}^m c_i y_i \\ \text{s.t.} \left\{ \begin{aligned} s_i^- x_i \leq y_i \leq s_i^+ x_i, i=1,2,\dots,m \\ \sum_{i=1}^m y_i = d \\ x_i = 0, 1, y_i \geq 0, i=1,2,\dots,m \end{aligned} \right. \end{aligned} \quad (2)$$

If demand is uncertain, the contract order quantity may be not equal to the actual demand. If $\sum_{i=1}^m y_i < d$, the enterprise needs to buy raw material from the

market, the cost higher than the supplier is r_1 . If $\sum_{i=1}^m y_i > d$, the excess part should be stored, increasing storage costs is r_2 .

The additional expenditure cost is determined by the demand, let z^+ is the part that the demand exceeds the contract order and z^- is the part that the demand is lower than the contract order. Apparently z^+ and z^- cannot be greater than 0, so there have:

$$\begin{aligned} \sum_{i=1}^m y_i + z^+ - z^- &= d \\ z^+ z^- &= 0 \end{aligned} \tag{3}$$

The additional expenditure cost is $r_1 z^+ + r_2 z^-$. According to the principles of goal programming, when the additional expenditure cost is the minimum, z^+ and z^- should not simultaneously be greater than zero, so the programming is:

$$\begin{aligned} \min & \sum_{i=1}^m c_i y_i + r_1 z^+ + r_2 z^- \\ \text{s.t.} & \begin{cases} s_i^- x_i \leq y_i \leq s_i^+ x_i, i=1,2,\dots,m \\ \sum_{i=1}^m y_i + z^+ - z^- = d \\ x_i = 0,1, y_i \geq 0, z^+, z^- \geq 0, i=1,2,\dots,m \end{cases} \end{aligned} \tag{4}$$

The programming is a parameter programming, and demand d is its parameters.

3. Absolute stability decision model of supplier selection problem

If demand is uncertain, the supplier selection model (10) is a parameter programming. Its optimal solutions under different demand are different. If the demand is a continuous parameter, it is to calculate the optimal solution for all parameters. Here assumed the demand is a discrete parameter, its value may be d_1, d_2, \dots, d_n . For each demand value, the programming (10)'s optimal value is v_1, v_2, \dots, v_n .

For a given decision (x, y, z) , it is stable to demand d_i if its objective function value is less than or equal to α times the optimal value under demand d_i . And demand d_i is called as its stability parameter. Absolute stability decision is a solution which stable range or the number of stable parameters is maximum (Ma, 2011).

In order to establish the mathematical model of absolute stability decision, it should identify the stability parameters of every solution. Lets the variable $u_l, l=1,2,\dots,n$ indicates whether the parameters d_l is stability parameter, if it is $u_l = 1$, else $u_l = 0$. So the total number of stability parameters is $\sum_{i=1}^n u_i$, and there:

$$u_l = \begin{cases} 1, & \text{if } \sum_{i=1}^m c_i y_i + r_1 z_i^+ + r_2 z_i^- \leq \alpha v_l, l = 1, 2, \dots, n. \\ 0, & \text{else} \end{cases} \quad (5)$$

It is equivalent to:

$$\sum_{i=1}^m c_i y_i + r_1 z_i^+ + r_2 z_i^- - \alpha v_l - M(1 - u_l) \leq 0, l = 1, 2, \dots, n. \quad (6)$$

Where M is a large enough positive number. The absolute stability model of supplier selection is:

$$\begin{aligned} & \max \sum_{i=1}^n u_i \\ & \text{s.t.} \begin{cases} s_i^- x_i \leq y_i \leq s_i^+ x_i^-, i = 1, 2, \dots, m \\ \sum_{i=1}^m y_i + z_i^+ - z_i^- = d_l, l = 1, 2, \dots, n \\ \sum_{i=1}^m c_i y_i + r_1 z_i^+ + r_2 z_i^- - \alpha v_l - M(1 - u_l) \leq 0, l = 1, 2, \dots, n \\ x_i = 0, 1, y_i \geq 0, i = 1, 2, \dots, m, \quad z^+, z^- \geq 0 \end{cases} \end{aligned} \quad (7)$$

This programming is an integer linear programming, it can be solved by branch and bound algorithm. α is its parameter, with its increase the number of stability parameters will increase.

4. Example

Consider there have 8 candidate enterprises, the maximum production capacity, minimum contract order quantity and unit transportation costs of each candidate enterprise are shown as Table 1.

Table 1: Production capacity of enterprises.

Enterprise	1	2	3	4	5	6	7	8
Maximum production capacity	4	5	4	3	6	5	6	4
Minimum contract order quantity	1.5	2	1.5	1	2	2	2	1.5
unit transportation cost	1.2	2.1	1.6	1.8	1.4	2.5	2.2	1.3

$r_1 = 1.75, r_2 = 1.2$, and it's the optimization model is:

$$\begin{aligned} & \min 1.2y_1 + 2.1y_2 + 1.6y_3 + 1.8y_4 + 1.4y_5 + 2.5y_6 + 2.2y_7 + 1.3y_8 + 1.75z^+ + 1.2z^- \\ & \text{s.t.} \begin{cases} y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + z^+ - z^- = d \\ y_1 - 4x_1 \leq 0, y_1 - 1.5x_1 \geq 0, y_2 - 5x_2 \leq 0, y_2 - 2x_2 \geq 0 \\ y_3 - 4x_3 \leq 0, y_3 - 1.5x_3 \geq 0, y_4 - 3x_4 \leq 0, y_4 - x_4 \geq 0 \\ y_5 - 6x_5 \leq 0, y_5 - 2x_5 \geq 0, y_6 - 5x_6 \leq 0, y_6 - 2x_6 \geq 0 \\ y_7 - 6x_7 \leq 0, y_7 - 2x_7 \geq 0, y_8 - 4x_8 \leq 0, y_8 - 1.5x_8 \geq 0 \\ x_i = 0, 1, y_i, z^+, z^- \geq 0, i = 1, 2, \dots, 8 \end{cases} \end{aligned} \tag{8}$$

The demand d is its parameter, and its value may be integer from 8 to 15. For all demand values, the programming's optimal values are shown as Table 2.

Table 2: Optimal value of demand.

Demand	8	9	10	11	12	13	14	15
Optimal values	10	11.5	12.8	14.2	15.6	17	18.4	20.15

Lets $\alpha = 1.056$ and $M = 30$, the absolutely stable model is:

$$\begin{aligned} & \max \sum_{l=1}^8 u_l \\ & \text{s.t.} \begin{cases} y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + z_l^+ - z_l^- = d_l, l = 1, 2, \dots, 8 \\ y_1 - 4x_1 \leq 0, y_1 - 1.5x_1 \geq 0, y_2 - 5x_2 \leq 0, y_2 - 2x_2 \geq 0 \\ y_3 - 4x_3 \leq 0, y_3 - 1.5x_3 \geq 0, y_4 - 3x_4 \leq 0, y_4 - x_4 \geq 0 \\ y_5 - 6x_5 \leq 0, y_5 - 2x_5 \geq 0, y_6 - 5x_6 \leq 0, y_6 - 2x_6 \geq 0 \\ y_7 - 6x_7 \leq 0, y_7 - 2x_7 \geq 0, y_8 - 4x_8 \leq 0, y_8 - 1.5x_8 \geq 0 \\ 1.2y_1 + 2.1y_2 + 1.6y_3 + 1.8y_4 + 1.4y_5 + 2.5y_6 + 2.2y_7 + 1.3y_8 \\ + 1.75z_l^+ + 1.2z_l^- - 1.056v_l - 30(1 - u_l) \leq 0, l = 1, 2, \dots, 8 \\ x_i = 0, 1, y_i, z_l^+, z_l^- \geq 0, i = 1, 2, \dots, 8, l = 1, 2, \dots, 8 \end{cases} \end{aligned} \tag{9}$$

The optimal solution of the program is to select Enterprise 1, 5 and 8 for suppliers, and it is stable on 7 cases of all potential demands.

5. Conclusion

The absolute stability decision model of supplier selection problem with uncertain demand was given. The model strikes a balance between optimality and stability of decision. It can be widely used in supply chain design and logistics management. The parameters of these stability decision models are discrete parameters. For continuous parameter, it can be solved by discretization of continuous demand.

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