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A New Multi-Attribute Group Decision-Making Approach Based on Fermatean Fuzzy Power Hamy Mean Operator and Novel Score Function

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Abstract. The topic of this paper is multi-attribute decision-making (MAGDM) under Fermatean fuzzy sets (FFSs). We studied MAGDM method form the perspective of aggregation operators. First, a novel score function of Fermatean fuzzy numbers (FFNs) is proposed, which is more powerful than existing score function. Second, new Fermatean fuzzy aggregation operators based on power average and Hamy mean operators were proposed. Our new introduced aggregation operators have the ability of reducing the bad effects of extreme evaluation values and capturing the interrelationship among multiple input FFNs. These characteristics make the proposed aggregation more suitable to handle Fermatean fuzzy MAGDM problems. Third, numerical experiments were conducted to illustrate the effectiveness of the new MAGDM method. Finally, comparative analysis was provided to further explain why decision makers should choose our method.

Keywords: Fermatean fuzzy sets, power average, Hamy mean, Fermatean fuzzy power Hamy mean operators, multi-attribute group decision-making.

1. Introduction

In the field of management sciences and operational research, multi-attribute group decision making (MAGDM) refers to a series of decision-making problems that aims to obtain the ranking orders of possible alternatives based on the evaluation valued that provided by decision makers (DMs) from multiple aspects (Hu et al., 2022; Krishnan et al., 2022; Naziris et al., 2022; Ning et al., 2022; Cai et al., 2022). There are many methodologies to solve MAGDM problems and one the convenient and explainable method is aggregation operator (AO). AO refers to a collection of special function that aggregate individual inputs into single ones. In MAGDM problems, AO is used to integrate attribute values to compute the overall evaluation values. By comparing the final comprehensive evaluation values, the ranking order of alternatives is obtained and the optimal one can be determined naturally. The weighted average and weighted geo-metric operators are two fundamental AOs that have been extensively used in daily life. However, in most practical MAGDM problems, there exits wide interrelationship between attributes, and such interrelationship between attributes should be considered. The Bonferroni mean (BM) (Bonferroni et al., 2011) is useful AOs, which is famous for its ability of capturing the interrelationship between aggregated variables. It is worth noting that BM was originated for crisp numbers. Nevertheless, we always encounter decision situations, which is fulfill with fuzziness and uncertainty. Hence, to make BM more practical, many scholars and scientists focused on extending the classical BM to different fuzzy environments, such as intuitionistic fuzzy sets (IFSs) (Xu et al., 2011), interval-valued IFSs (Xu et al.,2011), hesitant fuzzy sets (Xu et al., 2012), dual hesitant fuzzy sets (Tu et al., 2017), etc. Quite a few MAGDM methods based on BM have been proposed, however, their shortcoming is also evident, i.e., BM can only consider the interrelationship between any two arguments. When multiple attributes are interrelated, then BM is suitable to be used to compute the overall evaluation values. The Hamy mean (HM) (Hara et al., 1998) has recently drawn many scholars' interesting, owing to its capability of effectively dealing with the interrelationship among multiple inputs. Similar to BM, HM has been investigated to accommodate complicated and uncertain decision-making environments. Li et al. (Li et al., 2018) extended HM into IFSs to handle intuitionistic fuzzy MAGDM problems. Wu et al. (2019) studied HM under interval-valued IFSs to evaluate the competitiveness of tourist destinations. Liu and Liu (2019) proposed a HM based MAGDM method under linguistic intuitionistic fuzzy sets. Liu and You (2018) introduced a linguistic neutrosophic HM operator based decision-making method. Liu et al. (2019) used HM to select heal-care waste treatment technology under intuitionistic uncertain linguistic decision environments. For more applications of HM in MAGDM problems under different decision-making environments, we suggest authors to refer (Wu et al., 2018; Rong et al., 2020; Gulistan et al., 2018; Deng et al., 2018).

In (Liu et al., 2019), authors proposed a new hybrid AO, viz. the power Hamy mean (PHM) operator, which is a combination of HM with power average (PA) (Yager 2001) operator. The main motivation to propose PHM is that scholars have realized the importance of considering the interrelationship among attributes and simultaneously reducing the negative effect of unreasonable attribute values on the final decision results. The capability of PA in effectively dealing with unduly high or low input arguments has been widely recognized (Feng et al., 2020; Liu et al., 2000; Liu et al., 2019; Liao et al., 2018). PHM absorbs the advantages of PA and HM and hence, it is suitable and powerful to deal with actual-life decision-making problems. Afterwards, Liu et al. (2020) used PHM to aggregate normal wiggly hesitant linguistic information and applied the proposed AOs in the evaluation of land ecological security. Liu and Li (2020) generalized PHM into MAGDM problems with trapezoidal fuzzy two-dimensional linguistic evaluation information.

As a nonstandard fuzzy set theory, the Pythagorean fuzzy sets (PFSs) (Yager 2014), which target at over-coming the shortcoming of IFSs, can describe wider information span and more complicated decision-making situations. Therefore, they have been extensively applied in MAGDM problems and a few novel decision-making methods have been pro-posed. In (ma et al., 2016; Zhang et al., 2017; Liang et al., 2017; Wei et al., 2018; Li et al., 2018) scholars investigated PFSs based Pythagorean fuzzy AOs. In [35-38], the traditional decision-making methods, such as TOPSIS, TODIM, ELECTRE and MOORA, were extended PFSs some improved MAGDM methods were developed. In addition, extensions of PFSs and their applications in MAGDM problems are also an active research field, and decision-making methods based on interval-valued PFSs (Garg 2016), Pythagorean fuzzy uncertain linguistic sets (Liu et al., 2017), Pythagorean 2-tuple linguistic sets (Wei et al., 2020), interval-valued Pythagorean fuzzy linguistic sets (Du et al., 2017), dual hesitant Pythagorean fuzzy sets (Wei et al., 2017), 2-tuple linguistic Pythagorean fuzzy sets (Deng et al., 2018), etc.

However, PFSs still have limitations when expressing fuzzy decision-making information. A PFS A can be expressed as $A = \left(\mu_A(x), v_A(x)\right)$, then A should satisfy that the square sum of membership degree (MD) and non-membership degree (NMD) to be less than or equal to one, i.e., $\left(\mu_A(x)\right)^2 + \left(v_A(x)\right)^2 \leq 1$. It is worthy pointing out that this constraint cannot be always satisfied. Suppose $\alpha = (0.75,0.80)$to be an evaluation value that provided by DM, as $0.75^2 + 0.80^2 = 1.2015 > 1$, then α cannot be denoted by PFSs. Similar to proposing PFSs to overcome the drawback of IFSs, Senapati and Yager (2020) proposed the Fermatean fuzzy sets (FFS), with the constraint that cubed sum of MD and NMD degrees to be less than or equal to one. This characteristic makes FFSs more powerful and suitable to describe DMs' evaluation values in practical MAGDM issues. In (Senapati et al., 2019), authors proposed basic AOs for Fermatean fuzzy numbers (FFNs) and studied their applications in MAGDM. In addition, some scholars focused on extensions of FFSs and their applications in MAGDM. Liu et al. (2019) extended FFSs to Fermatean fuzzy linguistic sets and proposed Fermatean fuzzy linguistic TOPSIS method. Liu et al. (2019) continued to present a novel MAGDM method under Fermatean fuzzy linguistic sets based on linguistic scale function, TODIM and TOPSIS. Senapati and Yager (2019) proposed novel operations of FFNs and introduced Fermatean fuzzy weighted product model to deal with MAGDM problems.

Recently studies on FFSs based MAGDM methods are still not enough. The theory of Fermatean fuzzy AOs should be continued. The good performance of PHM has deeply impressed scholars; however, it has not been studied under FFSs, which is the first motivation of this paper. Second, existing score function of FFNs still has drawbacks and it is highly necessary to propose a new score function. Hence, in this paper we first put forward a new score function to rank FFNs. Second, we present novel AOs for FFNs, i.e., the Fermatean fuzzy PHM operator and Fermatean fuzzy power weighted HM operator, by extending PHM into FFSs. Finally, we present a novel MADM method under FFSs. The rest of this paper is organized as follows. Section 2 reviews some basic concepts and introduces a new score function for FFNs. Section 3 puts forward novel Fermatean fuzzy AOs and discusses their properties. Section 4 develops a novel method to MAGDM.

2. Basic Concepts

Some basic notions are briefly introduced in this section.

2.1. Fermatean fuzzy sets

Definition 1 (Senapati et al., 2020). Let X be an ordinary set, then a Fermatean fuzzy set (FFS) A is defined as follows

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle | x \in X\}, \tag{1}$$

where $\mu_A(x)$ and $v_A(x)$ denote the MD and NMD, such that $0 \leq \mu_A(x), v_A(x) \leq 1$ and $0 \leq \left(\mu_A(x)\right)^3 + \left(v_A(x)\right)^3 \leq 1$. In addition, $\pi_A(x) = \left(1 - \left(\mu_A(x)\right)^3 - \left(v_A(x)\right)^3\right)^{1/3}$ is called the indeterminacy degree. For convenience, we call the ordered pair $A = \left(\mu_A(x), v_A(x)\right)$ a Fermatean fuzzy number (FFN), which can be denoted as $\alpha = (\mu, v)$ for simplicity.

The basic operations of FFNs are defined as follows.

Definition 2 (Senapati et al., 2020). *Let* $\alpha = (\mu, v)$, $\alpha_1 = (\mu_1, v_1)$ *and* $\alpha_2 = (\mu_2, v_2)$ *are three FFNs, and* λ *be a positive real number, then*

(1) $\alpha_1 \oplus \alpha_2 = \left((\mu_1^3 + \mu_2^3 - \mu_1^3\mu_2^3)^{1/3}, v_1 v_2\right)$;

(2) $\alpha_1 \otimes \alpha_2 = \left(\mu_1\mu_2, (v_1^3 + v_2^3 - v_1^3 v_2^3)^{1/3}\right)$;

(3) $\lambda\alpha = \left(\left(1 - (1 - \mu^3)^\lambda\right)^{1/3}, v^\lambda\right)$;

(4) $\alpha^\lambda = \left(\mu^\lambda, \left(1 - (1 - v^3)^\lambda\right)^{1/3}\right)$.

The comparison method for FFNs is presented as follows.

Definition 3 (Senapati et al., 2020). Let $\alpha = (\mu, v)$ be an FFN, then the score function $S(\alpha)$ is defined as

$$S(\alpha) = \mu^3 - v^3, \tag{2}$$

and the accuracy function is expressed as

$$H(\alpha) = \mu^3 + v^3, \tag{3}$$

For two FFNs α_1 and α_2, then

If $S(\alpha_1) > S(\alpha_2)$, *then* $\alpha_1 > \alpha_2$;

If $S(\alpha_1) = S(\alpha_2)$, *then*

if $H(\alpha_1) > H(\alpha_2)$, *then* $\alpha_1 > \alpha_2$;

if $H(\alpha_1) = H(\alpha_2)$, *then* $\alpha_1 = \alpha_2$.

The normalized Hamming distance between any FFNs is defined as follows.

Definition 4. Let $\alpha_1 = (\mu_1, v_1)$ and $\alpha_2 = (\mu_2, v_2)$ be two FFNs, then the distance between α_1 and α_2 is defined as

$$d(\alpha_1, \alpha_2) = \frac{|\mu_1^3 - \mu_2^3| + |v_1^3 - v_2^3| + |\pi_1^3 - \pi_2^3|}{2}, \tag{4}$$

where π_1 and π_2 denote the indeterminacy degrees of α_1 and α_2, respectively.

2.2. Some basic aggregation operators

In this subsection, some basic aggregation operators that will be used in the followings are presented.

Definition 5 (Yager 2001). Let $a_i (i = 1,2, \dots, n)$ be a collection of non-negative crisp numbers, then the PA operator is defined as

$$PA(a_1, a_2, \dots, a_n) = \frac{\sum_{i=1}^{n}(1+T(a_i))a_i}{\sum_{i=1}^{n}(1+T(a_i))}, \tag{5}$$

where $T(a_i) = \sum_{j=1, i\neq j}^{n} Sup(a_i, a_j)$, $Sup(a_i, a_j)$ denotes the support for a_i from a_j, such that

(1) $0 \leq Sup(a_i, a_j) \leq 1$;

(2) $Sup(a_i, a_j) = Sup(a_j, a_i)$;

(3) $Sup(a, b) \leq Sup(c, d)$, *if and only if* $|a - b| \geq |c - d|$.

Definition 6 (Yager 2001). Let $a_i (i = 1,2, \dots, n)$ be a collection of crisp numbers and $k = 1,2, \dots, n$, the Hamy mean (HM) is expressed as

$$HM^{(k)}(a_1, \dots, a_n) = \frac{1}{C_n^k} \sum_{1 \leq i_1 < \cdots < i_k \leq n} \left(\prod_{j=1}^{k} a_{i_j}\right)^{1/k}, \tag{6}$$

where $(i_1, i_2, \dots, i_k)$ traverses all the k-tuple combination of $(1,2, \dots, n)$, and C_n^k is the binomial coefficient.

By combining PA with HM, Liu et al. [21] proposed the PHM operator, whose definition is presented as follows.

Definition 7 (Yager 2001). Let $a_i (i = 1,2, \dots, n)$ be a collection of crisp numbers and $k = 1,2, \dots, n$, the power Hamy mean (PHM) operator is defined as

$$PHM^{(k)}(a_1, \dots, a_n) = \frac{1}{C_n^k} \sum_{1 \le i_1 < \dots < i_k \le n} \left(\prod_{j=1}^k \left(\frac{n\left(1+T\left(a_{i_j}\right)\right)}{\sum_{i=1}^n (1+T(a_i))} a_{i_j} \right) \right)^{1/k}, \quad (7)$$

where $T(a_i) = \sum_{j=1, i \ne j}^n Sup\left(a_i, a_j\right)$, $Sup\left(a_i, a_j\right)$ denotes the support for a_i from a_j, satisfying the properties presented in Definition 8.

3. Novel Score Function and Corresponding Comparison Method of FFNs

In (Senapati et al., 2020), authors proposed score function and accuracy function of FFNs, and based on which a new ranking method for comparing FNNs was proposed. However, the score function and corresponding comparison method of FFNs still have shortcomings. We provide the following example to explain the drawbacks.

Example 1. Let $\alpha_1 = (0.6, 0.6)$ and $\alpha_2 = (0.7, 0.7)$ be any two FFNs, then according to Eq. (2), we have $S(\alpha_1) = S(\alpha_2) = 0$, which is somewhat counterintuitive. In addition, let $\beta_1 = (0.9, 0.1)$ and $\beta_2 = (0.1, 0.9)$ be any FFNs, then according to Eq. (3), we can obtain $H(\beta_1) = H(\beta_2) = 1$, which is also counterintuitive to a certain degree.

The main drawbacks of existing score function and accuracy function are that they only consider MD and NMD, but neglect hesitation degree, which leads to some unreasonable results. In order overcome such drawbacks, we propose a novel comprehensive score function of FFNs.

Definition 8. *Let $\alpha = (\mu, v)$ be an FFN, then the score function $S(\alpha)$ is defined as*

$$S(\alpha) = \mu^3 - v^3 + (\mu^3 - v^3)\pi^3, \quad (8)$$

Accordingly, a new comparison method for FFNs is developed.

Definition 9. *Let $\alpha_1 = (\mu_1, v_1)$ and $\alpha_2 = (\mu_2, v_2)$ be any two FFNs, then*

(1) *If $S(\alpha_1) > S(\alpha_2)$, then $\alpha_1 > \alpha_2$;*

(2) *If $S(\alpha_1) < S(\alpha_2)$, then $\alpha_1 < \alpha_2$;*

(3) *If $S(\alpha_1) = S(\alpha_2)$, then*

If $\pi_1 > \pi_2$, then $\alpha_1 < \alpha_2$;

If $\pi_1 = \pi_2$, then $\alpha_1 = \alpha_2$.

Additionally, the following theorem can be obtained.

Theorem 1. Let $\alpha = (\mu, v)$ be an FFN, then when μ increases then $S(\alpha)$ monotonically increases and when v increases then $S(\alpha)$ monotonically decreases.

In addition, it is easy to prove that the score function meets the following property.

Theorem 2. Let $\alpha = (\mu, v)$ be an FFN, then the score function $S(\alpha)$ meets:

(1) $0 \leq S(\alpha) \leq 1$;

(2) $S(\alpha) = 1$ iff $\alpha = (1,0)$;

(3) $S(\alpha) = -1$ iff $\alpha = (0,1)$.

4. Some Novel Aggregation Operators of FFNs

This section proposes some novel operators of FFNs and discusses their properties.

4.1. The Fermatean Fuzzy Power Average (FFPA) Operator

Definition 10. Let $\alpha_i = (\mu_i, v_i)(i = 1,2, \dots, n)$ be a collection of FFNs, then the Fermatean fuzzy power average (FFPHM) operator is expressed as

$$FFPA(\alpha_1, \alpha_2, \dots, \alpha_n) = \frac{\oplus_{i=1}^{n}(1+T(\alpha_i))\alpha_i}{\sum_{i=1}^{n}(1+T(\alpha_i))}. \tag{9}$$

where $T(\alpha_i) = \sum_{j=1, i\neq j}^{n} Sup(\alpha_i, \alpha_j)$, $Sup(\alpha_i, \alpha_j)$ denotes the support for α_i from α_j, such that

(1) $0 \leq Sup(\alpha_i, \alpha_j) \leq 1$;

(2) $Sup(\alpha_i, \alpha_j) = Sup(\alpha_j, \alpha_i)$;

(3) $Sup(\alpha_i, \alpha_j) \leq Sup(\alpha_s, \alpha_t)$, if and only if $d(\alpha_i, \alpha_j) \geq d(\alpha_s, \alpha_t)$.

If we assume that

$$\tau_i = \frac{1+T(\alpha_i)}{\sum_{i=1}^{n}(1+T(\alpha_i))}, \tag{10}$$

Then Eq. (10) can be written as

$$FFPA(\alpha_1, \alpha_2, \dots, \alpha_n) = \oplus_{i=1}^{n} \tau_i \alpha_i. \tag{11}$$

such that $0 \leq \tau_i \leq 1$ and $\sum_{i=1}^{n} \tau_i = 1$.

It is easy to prove the following theorems.

Theorem 3. *Let $\alpha_i = (\mu_i, v_i)(i = 1,2, \dots, n)$ be a collection of FFNs, then the aggregated value by using the FFPA is still a FFN and*

$$FFPA(\alpha_1, \alpha_2, \dots, \alpha_n) = \left(\left(1 - \prod_{i=1}^{n}\left(1 - \mu_i^{1/3}\right)^{\tau_i}\right)^{1/3}, \prod_{i=1}^{n} v_i^{\tau_i}\right). \tag{12}$$

Theorem 4 (**Idempotency**). Let $\alpha_i = (\mu_i, v_i)(i = 1,2, \dots, n)$ be a collection of FFNs, and $\alpha_i = \alpha = (\mu, v)$ for all i, then

$$FFPA(\alpha_1, \alpha_2, \dots, \alpha_n) = \alpha. \tag{13}$$

Theorem 5. (**Boundedness**). *Let $\alpha_i = (\mu_i, v_i)(i = 1,2, \dots, n)$ be a collection of FFNs, then*

$$\alpha^- \leq FFPA(\alpha_1, \alpha_2, \dots, \alpha_n) \leq \alpha^+, \tag{14}$$

where

$$\alpha^{+} = (max_{i=1}^{n}\mu_i, min_{i=1}^{n}v_i),$$

and

$$\alpha^{-} = (min_{i=1}^{n}\mu_i, max_{i=1}^{n}v_i).$$

4.2. The fermatean fuzzy power weighted average (FFPWA) operator

If the corresponding weight vector of aggregated FFNs is considered in the FFPA operator, then the weighted form is obtained, i.e., the Fermatean fuzzy power weighted average (FFPWA) operator.

Definition 11. Let $\alpha_i = (\mu_i, v_i)(i = 1,2, \dots, n)$ *be a collection of FFNs, and* $w = (w_1, w_2, \dots, w_n)^T$ *the associated weighted vector, such that* $0 \leq w_i \leq 1$ *and* $\sum_{i=1}^{n} w_i = 1$. *The Fermatean fuzzy power weighted average (FFPWA) operator is defined as*

$$FFPWA(\alpha_1, \alpha_2, \dots, \alpha_n) = \frac{\oplus_{i=1}^{n} w_i\left(1+T(\alpha_i)\right)\alpha_i}{\sum_{i=1}^{n} w_i\left(1+T(\alpha_i)\right)}, \tag{15}$$

where $T(\alpha_i) = \sum_{j=1, i\neq j}^{n} Sup\left(\alpha_i, \alpha_j\right)$, $Sup\left(\alpha_i, \alpha_j\right)$ denotes the support for α_i from α_j, satisfying the properties presented in Definition 10. We assume that

$$\varpi_i = \frac{w_i\left(1+T(\alpha_i)\right)}{\sum_{i=1}^{n} w_i\left(1+T(\alpha_i)\right)}, \tag{16}$$

then, Eq. (15) is written as

$$FFPWA(\alpha_1, \alpha_2, \dots, \alpha_n) = \oplus_{i=1}^{n} \varpi_i \alpha_i, \tag{17}$$

where $0 \leq \varpi_i \leq 1$ *and* $\sum_{i=1}^{n} \varpi_i = 1$.

Similarly, the FFPWA operator also has the following properties.

Theorem 6. Let $\alpha_i = (\mu_i, v_i)(i = 1,2, \dots, n)$ be a collection of FFNs, then the aggregated value by using the FFPWA is still a FFN and

$$FFPWA(\alpha_1, \alpha_2, \dots, \alpha_n) = \left(\left(1 - \prod_{i=1}^{n}\left(1 - \mu_i^{1/3}\right)^{\varpi_i}\right)^{1/3}, \prod_{i=1}^{n} v_i^{\varpi_i}\right). \tag{18}$$

Theorem 7 (**Idempotency**). Let $\alpha_i = (\mu_i, v_i)(i = 1,2, \dots, n)$ be a collection of FFNs, and $\alpha_i = \alpha = (\mu, v)$ for all i, then

$$FFPWA(\alpha_1, \alpha_2, \dots, \alpha_n) = \alpha. \tag{19}$$

Theorem 8 (**Boundedness**). *Let* $\alpha_i = (\mu_i, v_i)(i = 1,2, \dots, n)$ *be a collection of FFNs, then*

$$\alpha^{-} \leq FFPWA(\alpha_1, \alpha_2, \dots, \alpha_n) \leq \alpha^{+}, \tag{20}$$

where

$$\alpha^{+} = (max_{i=1}^{n}\mu_i, min_{i=1}^{n}v_i),$$

and

$$\alpha^- = (min_{i=1}^n \mu_i, max_{i=1}^n v_i).$$

4.3. The fermatean fuzzy power hamy mean (FFPHM) operator

This subsection proposes a combined AO for FFNs by extending the powerful PHM into FFSs. The detailed definition of the new operator is presented as follows.

Definition 12. Let $\alpha_i = (\mu_i, v_i)(i = 1,2, \dots, n)$ be a collection of FFNs and $k = 1,2, \dots, n$, then the Fermatean fuzzy power Hamy mean (FFPHM) operator is expressed as

$$FFPHM^{(k)}(\alpha_1, \alpha_2, \dots, \alpha_n) = \frac{1}{C_n^k} \oplus_{1 \le i_1 < \dots < i_k \le n} \left(\otimes_{j=1}^k \left(\frac{n\left(1+T\left(\alpha_{i_j}\right)\right)}{\sum_{i=1}^n (1+T(\alpha_i))} \alpha_{i_j} \right) \right)^{1/k}, \quad (21)$$

where $(i_1, i_2, \dots, i_k)$ traverses all the k-tuple combination of $(1,2, \dots, n)$, and C_n^k is the binomial coefficient. $T(\alpha_i) = \sum_{j=1,i \ne j}^n Sup(\alpha_i, \alpha_j)$, $Sup(\alpha_i, \alpha_j)$ denotes the support for α_i from α_j, satisfying the properties in Definition 10. We assume

$$\xi_i = \frac{1+T(\alpha_i)}{\sum_{i=1}^n (1+T(\alpha_i))}, \quad (22)$$

then Eq. (21) can be written as

$$FFPHM^{(k)}(\alpha_1, \alpha_2, \dots, \alpha_n) = \frac{1}{C_n^k} \oplus_{1 \le i_1 < \dots < i_k \le n} \left(\otimes_{j=1}^k \left(n\xi_{i_j} \alpha_{i_j} \right) \right)^{1/k}, \quad (23)$$

where $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ is called the power weight vector, such that $0 \le \xi_i \le 1$ and $\sum_{i=1}^n \xi_i = 1$.

Based on the operational rules of FFNs, we can obtain the following aggregated value of the FFPHM operator.

Theorem 9. Let $\alpha_i = (\mu_i, v_i)(i = 1,2, \dots, n)$ be a collection of FFNs, then the aggregated value by the FFPHM operator is also an FFN and

$$FFPHM^{(k)}(\alpha_1, \alpha_2, \dots, \alpha_n) = \left(\left(1 - \prod_{1 \le i_1 < \dots < i_k \le n} \left(1 - \prod_{j=1}^k \left(1 - \left(1 - \mu_{i_j}^3 \right)^{n\xi_{i_j}} \right)^{1/k} \right)^{1/C_n^k} \right)^{1/3}, \prod_{1 \le i_1 < \dots < i_k \le n} \left(1 - \prod_{j=1}^k \left(1 - v_{i_j}^{3n\xi_{i_j}} \right)^{1/k} \right)^{1/3C_n^k} \right). \quad (24)$$

Proof. According to definition 2, we have

$$n\xi_{i_j} \alpha_{i_j} = \left(\left(1 - \left(1 - \mu_{i_j}^3 \right)^{n\xi_{i_j}} \right)^{1/3}, v_{i_j}^{n\xi_{i_j}} \right),$$

and

$$\otimes_{j=1}^{k}\left(n\xi_{i_j}\alpha_{i_j}\right)=\left(\prod_{j=1}^{k}\left(1-\left(1-\mu_{i_j}^{3}\right)^{n\xi_{i_j}}\right)^{1/3},\left(1-\prod_{j=1}^{k}\left(1-v_{i_j}^{3n\xi_{i_j}}\right)\right)^{1/3}\right).$$

Hence, we can further obtain

$$\left(\otimes_{j=1}^{k}\left(n\xi_{i_j}\alpha_{i_j}\right)\right)^{1/k}=\left(\prod_{j=1}^{k}\left(1-\left(1-\mu_{i_j}^{3}\right)^{n\xi_{i_j}}\right)^{1/3k},\left(1-\prod_{j=1}^{k}\left(1-v_{i_j}^{3n\xi_{i_j}}\right)^{1/k}\right)^{1/3}\right)$$

and

$$\oplus_{1\le i_1<\cdots<i_k\le n}\left(\otimes_{j=1}^{k}\left(\frac{n\left(1+T\left(\alpha_{i_j}\right)\right)}{\sum_{i=1}^{n}\left(1+T(\alpha_i)\right)}\alpha_{i_j}\right)\right)^{1/k}=$$

$$\left(\left(1-\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}\left(1-\left(1-\mu_{i_j}^{3}\right)^{n\xi_{i_j}}\right)^{1/k}\right)\right)^{1/3},\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}\left(1-v_{i_j}^{3n\xi_{i_j}}\right)^{1/k}\right)^{1/3}\right)$$

Finally,

$$\frac{1}{C_n^k}\oplus_{1\le i_1<\cdots<i_k\le n}\left(\otimes_{j=1}^{k}\left(\frac{n\left(1+T\left(\alpha_{i_j}\right)\right)}{\sum_{i=1}^{n}\left(1+T(\alpha_i)\right)}\alpha_{i_j}\right)\right)^{1/k}=$$

$$\left(\left(1-\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}\left(1-\left(1-\mu_{i_j}^{3}\right)^{n\xi_{i_j}}\right)^{1/k}\right)^{1/C_n^k}\right)^{1/3},\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}\left(1-v_{i_j}^{3n\xi_{i_j}}\right)^{1/k}\right)^{1/3C_n^k}\right)$$

Additionally, the FFPHM operator has the following properties.

Theorem 10 (Idempotency). *Let $\alpha_i=(\mu_i,v_i)(i=1,2,\dots,n)$ be a collection of FFNs, and $\alpha_i=\alpha=(\mu,v)$ for all i, then*

$$FFPHM^{(k)}(\alpha_1,\alpha_2,\dots,\alpha_n)=\alpha. \tag{25}$$

Proof. As $\alpha_i=\alpha=(\mu,v)$ for i = 1,2, ..., n, then we can obtain $Sup\left(\alpha_i,\alpha_j\right)=1$ for $i,j=1,2,\dots,n$ and $\xi_i=1/n$. According to Theorem 9, we have

$FFPHM^{(k)}(\alpha_1,\alpha_2,\dots,\alpha_n)=FFPHM^{(k)}(\alpha,\alpha,\dots,\alpha)=$

$$\left(\left(1-\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}\left(1-(1-\mu^3)^{n\xi_{i_j}}\right)^{1/k}\right)^{1/C_n^k}\right)^{1/3},\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}\left(1-v^{3n\xi_{i_j}}\right)^{1/k}\right)^{1/3C_n^k}\right)$$

$$=(\mu,v)=\alpha \ \square$$

Theorem 11 (**Boundedness**). *Let $\alpha_i=(\mu_i,v_i)(i=1,2,\dots,n)$ be a collection of FFNs, $\alpha^-=\min(\alpha_1,\alpha_2,\dots,\alpha_n)$ and $\alpha^+=\max(\alpha_1,\alpha_2,\dots,\alpha_n)$, then*

$$x\le FFPHM(\alpha_1,\alpha_2,\dots,\alpha_n)\le y, \tag{20}$$

where

$$x=\frac{1}{C_n^k}\bigoplus_{1\le i_1<\cdots<i_k\le n}\left(\otimes_{j=1}^{k}\left(n\xi_{i_j}\alpha^-\right)\right)^{1/k},$$

and

$$y=\frac{1}{C_n^k}\bigoplus_{1\le i_1<\cdots<i_k\le n}\left(\otimes_{j=1}^{k}\left(n\xi_{i_j}\alpha^+\right)\right)^{1/k},$$

Proof. It is easy to prove that

$$n\xi_{i_j}\alpha^-\le n\xi_{i_j}\alpha_{ij},$$

and

$$\otimes_{j=1}^{k}\left(n\xi_{i_j}\alpha^-\right)\le\otimes_{j=1}^{k}\left(n\xi_{i_j}\alpha_{ij}\right)$$

Therefore,

$$\left(\otimes_{j=1}^{k}\left(n\xi_{i_j}\alpha^-\right)\right)^{1/k}\le\left(\otimes_{j=1}^{k}\left(n\xi_{i_j}\alpha_{ij}\right)\right)^{1/k},$$

and

$$\bigoplus_{1\le i_1<\cdots<i_k\le n}\left(\otimes_{j=1}^{k}\left(n\xi_{i_j}\alpha^-\right)\right)^{1/k}\le\bigoplus_{1\le i_1<\cdots<i_k\le n}\left(\otimes_{j=1}^{k}\left(n\xi_{i_j}\alpha_{i_j}\right)\right)^{1/k}$$

Finally,

$$\frac{1}{C_n^k}\bigoplus_{1\le i_1<\cdots<i_k\le n}\left(\otimes_{j=1}^{k}\left(n\xi_{i_j}\alpha^-\right)\right)^{1/k}\le\frac{1}{C_n^k}\bigoplus_{1\le i_1<\cdots<i_k\le n}\left(\otimes_{j=1}^{k}\left(n\xi_{i_j}\alpha_{i_j}\right)\right)^{1/k}$$

which implies that $x\le FFPHM(\alpha_1,\alpha_2,\dots,\alpha_n)$. Similarly, we can also prove that $FFPHM(\alpha_1,\alpha_2,\dots,\alpha_n)\le y$, which completes the proof of Theorem 11. □

In the following, we discuss some special cases of FFPHM operator with respect to the parameter *k*.

Case 1: *If $k=1$, the FFPHM operator is reduced to the Fermatean fuzzy power average (FFPA) operator, i.e.,*

$$FFPHM^{(1)}(\alpha_1,\alpha_2,\dots,\alpha_n)=\left(\left(1-\prod_{i=1}^{n}(1-\mu_i^3)^{\xi_i}\right)^{1/3},\prod_{i=1}^{n}v_i^{\xi_i}\right)=\bigoplus_{i=1}^{n}\xi_i\alpha_i=FFPA(\alpha_1,\alpha_2,\dots,\alpha_n). \tag{21}$$

In addition, if $Sup(\alpha_i, \alpha_j) = t > 0$, then FFPHM is reduced to Fermatean fuzzy average (FFA) operator, i.e.

$$FFPHM^{(1)}(\alpha_1, \alpha_2, \dots, \alpha_n) = \left(\left(1 - \prod_{i=1}^{n}(1-\mu_i^3)^{1/n}\right)^{1/3}, \prod_{i=1}^{n} v_i^{1/n}\right) = \frac{1}{n} \oplus_{i=1}^{n} \alpha_i = FFA(\alpha_1, \alpha_2, \dots, \alpha_n). \quad (22)$$

Case 2: If $k = n$, the FFPHM operator is reduced to the following form, i.e.,

$$FFPHM^{(n)}(\alpha_1, \alpha_2, \dots, \alpha_n) = \left(\prod_{i=1}^{n}\left(1-(1-\mu_i^3)^{n\xi_i}\right)^{\frac{1}{3n}}, \left(1-\prod_{i=1}^{n}\left(1-v_i^{3n\xi_i}\right)^{1/n}\right)^{1/3}\right) = (\otimes_{i=1}^{n} n\xi_i\alpha_i)^{1/n}, \quad (23)$$

In addition, if $Sup(\alpha_i, \alpha_j) = t > 0$, then FFPHM is reduced to Fermatean fuzzy geometric (FFG) operator, i.e.

$$FFPHM^{(n)}(\alpha_1, \alpha_2, \dots, \alpha_n) = \left(\prod_{i=1}^{n} \mu_i^{1/n}, \left(1-\prod_{i=1}^{n}(1-v_i^3)^{1/n}\right)^{1/3}\right) = \otimes_{i=1}^{n} \alpha_i^{1/n} = FFG(\alpha_1, \alpha_2, \dots, \alpha_n). \quad (24)$$

4.4. The fermatean fuzzy power weighted hamy mean (FFPWHM) operator

If the weights of aggregated FFNs are taken into consideration in the FFPHM operator, then the Fermatean fuzzy power weighted Hamy mean operator is derived.

Definition 14. Let $\alpha_i = (\mu_i, v_i)(i = 1,2, \dots, n)$ be a collection of FFNs and $k = 1,2, \dots, n$. Let $w = (w_1, w_2, \dots, w_n)^T$ be the corresponding weight vector, such that $\sum_{i=1}^{n} w_i = 1$ and $0 \le w_i \le 1$. The 3.4. The Fermatean fuzzy power weighted Hamy mean (FFPWHM) operator is expressed as

$$FFPWHM^{(k)}(\alpha_1, \alpha_2, \dots, \alpha_n) = \frac{1}{C_n^k} \oplus_{1 \le i_1 < \dots < i_k \le n} \left(\otimes_{j=1}^{k}\left(\frac{nw_{i_j}\left(1+T\left(\alpha_{i_j}\right)\right)}{\sum_{i=1}^{n} w_{i_j}\left(1+T(\alpha_i)\right)}\alpha_{i_j}\right)\right)^{1/k}, \quad (25)$$

where $(i_1, i_2, \dots, i_k)$ traverses all the k-tuple combination of $(1,2, \dots, n)$, and C_n^k is the binomial coefficient. $T(\alpha_i) = \sum_{j=1, i \ne j}^{n} Sup(\alpha_i, \alpha_j)$, $Sup(\alpha_i, \alpha_j)$ denotes the support for α_i from α_j, satisfying the properties in Definition 11. If

$$\eta_i = \frac{w_i\left(1+T(\alpha_i)\right)}{\sum_{i=1}^{n} w_i\left(1+T(\alpha_i)\right)}, \quad (26)$$

then Eq. (25) can be transformed into

$$FFPWHM^{(k)}(\alpha_1, \alpha_2, \dots, \alpha_n) = \frac{1}{C_n^k} \oplus_{1 \le i_1 < \dots < i_k \le n} \left(\otimes_{j=1}^{k}\left(n\eta_{i_j}\alpha_{i_j}\right)\right)^{1/k}, \quad (27)$$

where $\eta = (\eta_1, \eta_2, \dots, \eta_n)^T$ is called the power weight vector, such that $0 \le \eta_i \le 1$ and $\sum_{i=1}^{n} \eta_i = 1$.

Theorem 12. Let $\alpha_i = (\mu_i, v_i)(i = 1,2,\dots,n)$ be a collection of FFNs, the aggregated result by the FFPWHM operator is still an FFNs and

$$FFPWHM^{(k)}(\alpha_1,\alpha_2,\dots,\alpha_n) = \left(\left(1-\prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}\left(1-\left(1-\mu_{i_j}^3\right)^{n\xi_{i_j}}\right)^{1/k}\right)^{1/C_n^k}\right)^{1/3}, \prod_{1\le i_1<\cdots<i_k\le n}\left(1-\prod_{j=1}^{k}\left(1-v_{i_j}^{3n\xi_{i_j}}\right)^{1/k}\right)^{1/3C_n^k}\right). \quad (28)$$

The proof of Theorem 12 is similar to that of Theorem 9. Moreover, the FFPWHM operator also has the following properties.

Theorem 13 (**Idempotency**). *Let* $\alpha_i = (\mu_i, v_i)(i = 1,2,\dots,n)$ *be a collection of FFNs, and* $\alpha_i = \alpha = (\mu, v)$ *for all i, then*

$$FFPWHM^{(k)}(\alpha_1,\alpha_2,\dots,\alpha_n) = \alpha. \quad (28)$$

Theorem 14 (**Boundedness**). *Let* $\alpha_i = (\mu_i, v_i)(i = 1,2,\dots,n)$ *be a collection of FFNs, then*

$$\alpha^- \le FFPWHM^{(k)}(\alpha_1,\alpha_2,\dots,\alpha_n) \le \alpha^+, \quad (29)$$

where

$$\alpha^+ = (max_{i=1}^n \mu_i, min_{i=1}^n v_i),$$

and

$$\alpha^- = (min_{i=1}^n \mu_i, max_{i=1}^n v_i).$$

5. A novel approach to MAGDM based on the proposed operators

In this section, with the help of the proposed AOs, we propose a new MAGDM method under FFSs. Let $A = \{A_1, A_2, \dots, A_m\}$ be a set of candidates, and $G = \{G_1, G_2, \dots, G_n\}$ be a collective of attributes, whose attribute vector is $w = (w_1, w_2, \dots, w_n)^T$, such that $\sum_{j=1}^n w_j = 1$ and $0 \le w_j \le 1$. A group of t DMs $(D_1, D_2, \dots, D_t)$ are invited to evaluate the n alternatives. DMs are kindly invited to express their evaluation values and specifically for alternative $A_i(i = 1,2,\dots,m)$ of attribute $G_j(j = 1,2,\dots,n)$. A FFN $\alpha_{ij}^{(l)} = \left(\mu_{ij}^{(l)}, v_{ij}^{(l)}\right)$ is used to describe the evaluate value of attribute G_j of alternative X_i that provided by DM $D_l(l = 1,2,\dots,t)$. Weight vector of the group of DMs is $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_t)^T$, such that $\sum_{l=1}^t \lambda_t = 1$ and $0 \le \lambda_t \le 1$. Finally, t Fermatean fuzzy decision matrices are obtained. In the following, a novel MAGDM method with FFNs is developed.

Step 1. The original decision matrices should be normalized according to the following equation

$$\alpha_{ij}^{(l)} = \begin{cases} \left(\mu_{ij}^{(l)}, v_{ij}^{(l)}\right) & if\ G_j\ is\ benefit\ type \\ \left(v_{ij}^{(l)}, \mu_{ij}^{(l)}\right) & if\ G_j\ is\ cost\ type \end{cases}, \tag{30}$$

Step 2. Calculate the supports for $\alpha_{ij}^{(a)}$ from $\alpha_{ij}^{(b)}(a, b = 1,2, \dots t; a \neq b)$, i.e., $Sup\left(\alpha_{ij}^{(a)}, \alpha_{ij}^{(b)}\right)$ by

$$Sup\left(\alpha_{ij}^{(a)}, \alpha_{ij}^{(b)}\right) = 1 - d\left(\alpha_{ij}^{(a)}, \alpha_{ij}^{(b)}\right), \tag{31}$$

wherein $d\left(\alpha_{ij}^{(a)}, \alpha_{ij}^{(b)}\right)$ denotes the distance between $\alpha_{ij}^{(a)}$ and $\alpha_{ij}^{(b)}$, and its definition can be found in Definition 4.

Step 3. Calculate the overall support for $\alpha_{ij}^{(a)}$, i.e., $T\left(\alpha_{ij}^{(a)}\right)$ according to the following equation

$$T\left(\alpha_{ij}^{(a)}\right) = \sum_{a,b=1;a\neq b}^{t} Sup\left(\alpha_{ij}^{(a)}, \alpha_{ij}^{(b)}\right). \tag{32}$$

Step 4. Determine the power weight $\eta_{ij}^{(a)}$ associated with $\alpha_{ij}^{(a)}$, according to

$$\eta_{ij}^{(a)} = \frac{\lambda_a\left(1+T\left(\alpha_{ij}^{(a)}\right)\right)}{\sum_{a=1}^{t}\lambda_a\left(1+T\left(\alpha_{ij}^{(a)}\right)\right)}, \tag{33}$$

where obviously $0 \leq \eta_{ij}^{(a)} \leq 1$ *and* $\sum_{a=1}^{t}\eta_{ij}^{(a)} = 1$.

Step 5. Compute the overall evaluation value α_{ij} *of attribute* $G_j(j = 1,2, \dots, n)$ *of alternative* $A_i(i = 1,2, \dots, m)$ *by*

$$\alpha_{ij} = FFPWA\left(\alpha_{ij}^{(1)}, \alpha_{ij}^{(2)}, \dots, \alpha_{ij}^{(t)}\right), \tag{34}$$

Step 6. Calculate the supports for α_{ie} from $\alpha_{if}(e, f = 1,2 \dots, n; e \neq f)$ by

$$Sup\left(\alpha_{ie}, \alpha_{if}\right) = 1 - d\left(\alpha_{ie}, \alpha_{if}\right), \tag{35}$$

where $d\left(\alpha_{ie}, \alpha_{if}\right)$ denotes the distance between α_{ie} and α_{if}

Step 7. Compute the overall support for α_{ie} by

$$T(\alpha_{ie}) = \sum_{e,f=1;e\neq f}^{n} Sup\left(\alpha_{ie}, \alpha_{if}\right), \tag{36}$$

Step 8. Determine the overall weight ξ_{ie} associated with α_{ie} by

$$\xi_{ie} = \frac{w_e\left(1+T(\alpha_{ie})\right)}{\sum_{e=1}^{n} w_e\left(1+T(\alpha_{ie})\right)}, \tag{37}$$

wherein $0 \leq \xi_{ie} \leq 1$ and $\sum_{e=1}^{n}\xi_{ie} = 1$.

Step 9. Calculate the overall evaluation value α_i of alternative $A_i(i = 1,2, \dots, m)$ according to

$$\alpha_i = FFPWHM^{(k)}(\alpha_{i1}, \alpha_{i2}, \dots, \alpha_{in}), \tag{38}$$

Step 10. Calculate the score values $S(\alpha_i)$ of α_i.

Step 11. Determined the ranking order to alternatives based on their score values.

6. Illustrative Example

Example 2. Let's consider a chip supplier selection problem. Modern chip industry plays a vital role in the development of national economy. The development of computer, integrated circuit, cloud computing, automobile, home appliance and other industries all depend on the development of chip industry. Continuous and stable chip supply is very important for the development of enterprises. Therefore, the choice of chip suppliers is very important for the survival and development of enterprises. The choice of chip suppliers is essentially a MAGDM problem. A laptop manufacturer plans to choose one of the four chip suppliers for long-term cooperation. The four chip suppliers are denoted by$A = \{A_1, A_2, A_3, A_4\}$. The laptop manufacturer invites three experts (D_1, D_2, and D_3) to evaluate performance of the four candidate suppliers. The weight vector of the three experts is $\lambda = (0.35,0.3,0.3)^T$. To evaluate the performance of the four alternatives, experts are required to evaluate the four alternatives from four attributes, i.e., reliability (G_1), market share (G_2), product competitiveness (G_3), and quality of service (G_4). Weight vector of the four attributes is $w = (0.3,0.2,0.2,0.3)^T$. DMs are required to use FFNs to describe their evaluation values and three original Fermatean fuzzy decision matrices are obtained, which are listed in Tables 1-3.

Table 1: The original fermatean fuzzy decision matrix provided D_1.

	G_1	G_2	G_3	G_4
A_1	(0.2,0.3)	(0.3,0.4)	(0.5,0.7)	(0.7,0.8)
A_2	(0.4,0.6)	(0.5,0.6)	(0.6,0.7)	(0.3,0.5)
A_3	(0.2,0.4)	(0.5,0.6)	(0.3,0.5)	(0.6,0.7)
A_4	(0.4,0.7)	(0.3,0.6)	(0.6,0.8)	(0.2,0.5)

Table 2: The original Fermatean fuzzy decision matrix provided D_2.

	G_1	G_2	G_3	G_4
A_1	(0.4,0.5)	(0.4,0.6)	(0.3,0.5)	(0.4,0.6)
A_2	(0.3,0.7)	(0.4,0.6)	(0.5,0.6)	(0.4,0.3)
A_3	(0.1,0.4)	(0.6,0.7)	(0.2,0.5)	(0.3,0.5)
A_4	(0.6,0.7)	(0.4,0.6)	(0.3,0.5)	(0.3,0.5)

Table 3: The original Fermatean fuzzy decision matrix provided D_3.

	G_1	G_2	G_3	G_4
A_1	(0.4,0.6)	(0.4,0.5)	(0.5,0.7)	(0.2,0.5)
A_2	(0.3,0.5)	(0.4,0.5)	(0.7,0.8)	(0.3,0.6)
A_3	(0.4,0.6)	(0.4,0.5)	(0.5,0.6)	(0.6,0.7)
A_4	(0.3,0.5)	(0.2,0.4)	(0.5,0.6)	(0.6,0.7)

6.1. The decision-making process

Step 1. As all attributes are benefit type the original decision matrices do not need to be normalized.

Step 2. Calculate the supports for $\alpha_{ij}^{(a)}$ from $\alpha_{ij}^{(b)}(a, b = 1,2, \dots t; a \neq b)$, i.e., $Sup\left(\alpha_{ij}^{(a)}, \alpha_{ij}^{(b)}\right)$ according to Eq. (31), we can obtain the following results. For the convenient description, we use the symbol $Sup^{(a,b)}$ to denote the values of $Sup\left(\alpha_{ij}^{(a)}, \alpha_{ij}^{(b)}\right)$.

$$Sup^{(1,2)} = Sup^{(2,1)} = \begin{bmatrix} 0.9614 & 0.8110 & 0.6840 & 0.4250 \\ 0.8730 & 0.9390 & 0.7820 & 0.9020 \\ 0.9930 & 0.7820 & 0.9810 & 0.5930 \\ 0.8480 & 0.9630 & 0.4240 & 0.9810 \end{bmatrix}$$

$$Sup^{(1,3)} = Sup^{(3,1)} = \begin{bmatrix} 0.7550 & 0.9020 & 1.0000 & 0.2780 \\ 0.8720 & 0.8480 & 0.7040 & 0.9090 \\ 0.7920 & 0.8480 & 0.8110 & 1.0000 \\ 0.7450 & 0.8290 & 0.6130 & 0.5740 \end{bmatrix}$$

$$Sup^{(2,3)} = Sup^{(3,2)} = \begin{bmatrix} 0.9090 & 0.9090 & 0.6840 & 0.8530 \\ 0.7820 & 0.9090 & 0.4860 & 0.8110 \\ 0.7850 & 0.6300 & 0.7920 & 0.5930 \\ 0.5930 & 0.7920 & 0.8110 & 0.5930 \end{bmatrix}$$

Step 3. Calculate the overall support for $\alpha_{ij}^{(a)}$, i.e., $T\left(\alpha_{ij}^{(a)}\right)$ according to Eq. (32), we can obtain the following results. We use the symbol $T^{(a)}$ to denote the values $T\left(\alpha_{ij}^{(a)}\right)$.

$$T^{(1)} = \begin{bmatrix} 1.6010 & 1.7130 & 1.6840 & 1.7030 \\ 1.7450 & 1.7870 & 1.4860 & 1.8110 \\ 1.7850 & 1.6300 & 1.7920 & 1.5930 \\ 1.5930 & 1.7920 & 1.0370 & 1.5550 \end{bmatrix}$$

$$T^{(2)} = \begin{bmatrix} 1.7550 & 1.7200 & 1.3680 & 1.2780 \\ 1.6550 & 1.8480 & 1.2680 & 1.7130 \\ 1.7780 & 1.4120 & 1.7730 & 1.1860 \\ 1.4410 & 1.7550 & 1.2350 & 1.5740 \end{bmatrix}$$

$$T^{(3)} = \begin{bmatrix} 1.6640 & 1.8110 & 1.6840 & 1.1310 \\ 1.6640 & 1.7570 & 1.1900 & 1.7200 \\ 1.5770 & 1.4780 & 1.6030 & 1.5930 \\ 1.3380 & 1.6210 & 1.4240 & 1.1670 \end{bmatrix}$$

Step 4. Calculate the power weight $\eta_{ij}^{(a)}$ associated with $\alpha_{ij}^{(a)}$, according to Eq. (33), we can obtain the following results. Similarly, we use the symbol $\eta^{(a)}$ to denote the values of $\eta_{ij}^{(a)}$.

$$\eta^{(1)} = \begin{bmatrix} 0.3405 & 0.3459 & 0.3650 & 0.2932 \\ 0.3577 & 0.3485 & 0.3749 & 0.3578 \\ 0.3583 & 0.3670 & 0.3581 & 0.3703 \\ 0.3684 & 0.3582 & 0.3208 & 0.3657 \end{bmatrix}$$

$$\eta^{(2)} = \begin{bmatrix} 0.3606 & 0.3468 & 0.3221 & 0.3922 \\ 0.3459 & 0.3561 & 0.3420 & 0.3454 \\ 0.3574 & 0.3366 & 0.3557 & 0.3122 \\ 0.3468 & 0.3535 & 0.3520 & 0.3684 \end{bmatrix}$$

$$\eta^{(3)} = \begin{bmatrix} 0.2989 & 0.3072 & 0.3129 & 0.3145 \\ 0.2964 & 0.2955 & 0.2811 & 0.2986 \\ 0.2842 & 0.2964 & 0.2862 & 0.3174 \\ 0.2847 & 0.2883 & 0.3272 & 0.2659 \end{bmatrix}$$

Step 5. Calculate the overall evaluation value α_{ij} of attribute $G_j(j = 1,2, \ldots, n)$ of alternative $A_i(i = 1,2, \ldots, m)$ according to Eq. (34), we can obtain the comprehensive decision matrix as shown in Table 4.

Table 4: The comprehensive decision matrix.

	***G*₁**	***G*₂**	***G*₃**	***G*₄**
A_1	(0.3565,0.4437)	(0.3717,0.4931)	(0.4556,0.6281)	(0.5201,0.6164)
A_2	(0.3431,0.5996)	(0.4409,0.5685)	(0.6089,0.6896)	(0.3418,0.4424)
A_3	(0.2793,0.4489)	(0.5189,0.5987)	(0.3674,0.5268)	(0.5444,0.6302)
A_4	(0.4792,0.6360)	(0.3267,0.5338)	(0.4975,0.6171)	(0.4213,0.5468)

Step 6. Calculate the supports for α_{ie} from $\alpha_{if}(e,f = 1,2 \ldots, n; e \neq f)$ according to Eq. (35), we can obtain that

$$Sup^{(1,2)} = Sup^{(2,1)} = (0.9614,0.9547,0.7579,0.8197);$$
$$Sup^{(1,3)} = Sup^{(1,3)} = (0.7903,0.7022,0.9164,0.9777);$$
$$Sup^{(1,4)} = Sup^{(4,1)} = (0.7577,0.8706,0.7006,0.8709);$$
$$Sup^{(2,3)} = Sup^{(3,2)} = (0.8289,0.7158,0.8414,0.8289);$$
$$Sup^{(2,4)} = Sup^{(4,2)} = (0.7963,0.8571,0.9428,0.9487);$$
$$Sup^{(3,4)} = Sup^{(4,3)} = (0.9539,0.5729,0.7842,0.8801).$$

Step 7. Compute the overall support for α_{ie} according to Eq. (36), we can obtain that

$$T = \begin{bmatrix} 2.5094 & 2.5866 & 2.5730 & 2.5079 \\ 2.5275 & 2.5275 & 1.9909 & 2.3005 \\ 2.3750 & 2.5421 & 2.5421 & 2.4276 \\ 2.6683 & 2.5973 & 2.6867 & 2.6998 \end{bmatrix}$$

Step 8. Calculate the overall weight ξ_{ie} associated with α_{ie} according to Eq. (37), we can obtain that

$$\xi = \begin{bmatrix} 0.2977 & 0.2028 & 0.2020 & 0.2975 \\ 0.3157 & 0.2105 & 0.1784 & 0.2954 \\ 0.2928 & 0.2049 & 0.2049 & 0.2974 \\ 0.3001 & 0.1962 & 0.2011 & 0.3027 \end{bmatrix}$$

Step 9. Calculate the overall evaluation value α_i of alternative $A_i(i = 1,2, \ldots, m)$ according to Eq. (38) (suppose that $k = 3$), we can obtain that

$$\alpha_1 = (0.4204,0.5708);\ \alpha_2 = (0.4206,0.6120)$$
$$\alpha_3 = (0.4155,0.5722);\ \alpha_4 = (0.4257,0.5977)$$

Step 10. Calculate the score values $S(\alpha_i)$ of alternative $A_i(i = 1,2, \ldots, m)$, we can obtain that

$$S(\alpha_1) = -0.2330, S(\alpha_2) = -0.2993, S(\alpha_3) = -0.2422, S(\alpha_4) = -0.2687$$

Step 11. We can obtain the ranking order of alternatives, i.e. ,$A_1 \succ A_3 \succ A_4 \succ A_2$. Hence, A_1 is the best alternative.

6.2. Analysis of the influence of the parameter k on the decision results

This subsection analyzes the impact of the influence of the parameter k on the final decision results. We assign different values to k in the FFPWHM operator and corresponding ranking orders of alternatives are obtained. To better demonstrate the change tendency of the score values, Figure 1 is provided as follows.

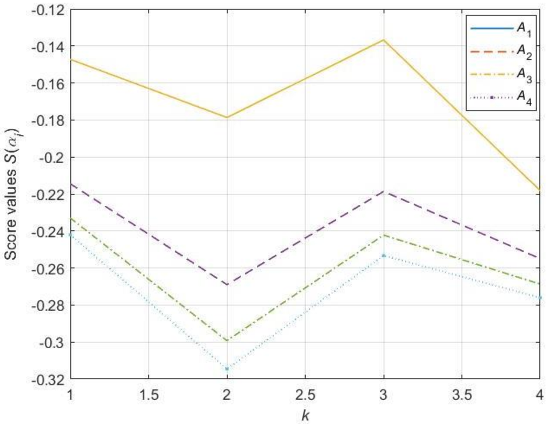

Fig. 1: Score values of ranking orders of alternatives.

As seen from Figure 1, when different values of k are employed, different score values of alternatives are obtained. However, the ranking orders of alternatives are always $A_1 \succ A_3 \succ A_4 \succ A_2$ and A_1 also indicates the robustness and stability of our proposed MAGDM method. As a matter of factor, the parameter k plays an important role in the information aggregation process. If there is no interrelationship between attributes, i.e., all attributes are independent, we can set $k = 1$. If the relationship exists between two attributes, we can set $k = 2$. In addition, we can set $k = 4$ if all the four attributes are interrelated. DMs can select an appropriate value of k according to actual decision-making situations.

6.3. Comparision with some existing MAGDM methods

In this section, to illustrate the advantages and superiorities of the proposed method, we compare our proposed method with that developed by Senapati and Yager (2019) based on the Fermatean fuzzy weighted averaging (FFWA) operator, and that presented by Gao et al. (2019) based on the intuitionistic fuzzy power Maclaurin symmetric mean (IFPWMSM) operator. Details of the comparison are provided as follows.

6.3.1. Compared with method developed by Senapati and Yager [49]

To compare our method with that proposed by Senapati and Yager (2019), we utilize these two methods to solve the following Example 3.

Example 3 (Revised from Senapati and Yager (2019)). Suppose that a group of Professors of one university wants to choose a suitable place to construct their home.

They select four places, i.e., Ashoke Nagar (A_1), Judge Court (A_2), Patna Bazar (A_3) and Kshudiram Nagar (A_4), as the alternatives within 10 Kilometers of their university. After consultation, they set five criteria , i.e., lifestyle & neighbors (G_1), soil type (G_2), size, shape, orientation, and slope of the block of land (G_3), existing roads and access to essential services (G_4), and cost (G_5) for choosing the most suitable place for home construction. After deliberation, the weight vector of criteria $w = (0.2,0.2,0.1,0.3,0.2)^T$ is adopted with unanimous consent. Suppose that the evaluation values of the alternatives with respect to each criterion are represented by Fermatean fuzzy numbers, and the original decision matrix is shown in Table 6. It should be noted that the criterion cost (G_5) is a negative attribute and should be normalized in the calculation process. DMs' evaluation values are in the form of FFNs, which are listed in Table 5. We employ our proposed method based on FFPWHM operator and Senapati and Yager's (2019) method based on FFWA operator to handle this example, and the calculation results are shown in Table 6.

Table 5: The Fermatean fuzzy decision matrix of Example 3.

	G_1	G_2	G_3	G_4	G_5
A_1	(0.7,0.3)	(0.4,0.6)	(0.5,0.5)	(0.8,0.2)	(0.8,0.4)
A_2	(0.5,0.8)	(0.8,0.6)	(0.4,0.5)	(0.7,0.4)	(0.6,0.5)
A_3	(0.9,0.6)	(0.8,0.1)	(0.6,0.4)	(0.7,0.5)	(0.9,0.3)
A_4	(0.6,0.7)	(0.8,0.3)	(0.7,0.2)	(0.5,0.3)	(0.7,0.3)

Table 6: The calculation results of Example 3 by dealing with different methods.

Methods	Score values $S(\alpha_i)$	Ranking orders
Senapati and Yager's [49] method	$S(\alpha_1) = -0.1143, S(\alpha_2) = -0.0418,$ $S(\alpha_3) = -0.1681, S(\alpha_4) = -0.0845$	$A_2 \succ A_4 \succ A_1 \succ A_3$
Our proposed method ($k = 3$)	$S(\alpha_1) = -0.1013, S(\alpha_2) = -0.1321,$ $S(\alpha_3) = -0.0150, S(\alpha_4) = -0.0402$	$A_3 \succ A_4 \succ A_1 \succ A_2$

As it is seen from Table 6 that both our proposed method and Senapati and Yager's (2019) method can solve this example, which also indicates the effectiveness of our proposed method. In addition, the decision results produced by the two methods are different. The ranking order derived by Senapati and Yager's (2019) method is $A_2 \succ A_4 \succ A_1 \succ A_3$ and A_2 is the best alternative. Our proposed method produces $A_3 \succ A_4 \succ A_1 \succ A_2$ and A_3 is the optimal alternative. Both the two methods are effective to handle real MAGDM problems; however, our method is more powerful and flexible than the proposed by Senapati and Yager's (2019). First of all, our method is based on the PA operator, so it is effective to felicitously handle unreasonable evaluation values. Second, our method can effectively deal with the interrelationship among attributes. However, Senapati and Yager's (2019) method is based on the simple weighted average operator. Compared with our method, the drawbacks of Senapati and Yager's (2019) decision-making method are obvious. First, it is powerless to cope with DMs' unduly high or low evaluation values. In other words, if DMs provide some extreme evaluation values, then Senapati and Yager's (2019) method would also produce unreasonable decision results. Second, Senapati and Yager's (2019) method does not consider the interrelationship among attributes. In other words, Senapati and Yager's (2019) method only considers decision-making problems where attributes are independent. However, in most MAGDM problems, attributes and interrelated and the interrelationship among them should be taken into account when determining the final decision results. Hence, compared with Senapati and Yager's (2019) method, our proposed method is more suitable to handle realistic decision making problems.

6.3.2. Compared with the method developed by Gao et al. (2019)

To compare our method with Gao et al.' (2019) method, we provide the following example. We use the two methods to solve the following example and discuss their decision results.

Example 4 (Revised from Gao et al. (2019)). In order to increase the value of customer experience and expand the market, a bike-sharing operating company decides to put some new types of sharing bikes into the market. At the moment, there are four different types of bicycles made by four different manufactures, and it hard to decide which one is the best. Thus, the manager decides to invite a tester to give his/her evaluation by testing them personally. Suppose that (A_1, A_2, A_3, A_4) are utilized to represent the four types, and the tester is required to evaluate the bike with respect to four attributes, containing safety (G_1), comfortability (G_2), convenience (G_3) and aesthetic (G_4). The weight vector of the four attributes is$w = (0.4,0.3,0.2,0.1)^T$. The manager requires the tester to provide his/her evaluation with intuitionistic fuzzy numbers, and the decision matrix is shown in Table 7. To compare our method with the method based on IFPWMSM operator, we utilize these two methods to solve this example simultaneously, and the comparison results are shown in Table 8.

Table 7: The intuitionistic fuzzy decision matrix of Example 4.

	G_1	G_2	G_3	G_4
A_1	(0.6,0.1)	(0.7,0.3)	(0.7,0.1)	(0.4,0.3)
A_2	(0.7,0.2)	(0.6,0.1)	(0.5,0.4)	(0.5,0.3)
A_3	(0.3,0.3)	(0.6,0.2)	(0.7,0.2)	(0.6,0.1)
A_4	(0.6,0.3)	(0.5,0.2)	(0.4,0.4)	(0.5,0.3)

Table 8: The calculation results of example 4 by using different methods.

Methods	Score values $S(\alpha_i)$	Ranking orders
Gao et al.'s [50] method (suppose that $k = 2$)	$S(\alpha_1) = -0.2173, S(\alpha_2) = -0.1823,$ $S(\alpha_3) = -0.1800, S(\alpha_4) = -0.1485$	$A_1 \succ A_2 \succ A_3 \succ A_4$
Our proposed method (suppose that $k = 2$)	$S(\alpha_1) = 0.4575, S(\alpha_2) = 0.3507,$ $S(\alpha_3) = 0.3887, S(\alpha_4) = 0.1657$	$A_1 \succ A_3 \succ A_2 \succ A_4$

As it is seen from Table 8, both Gao et al.'s (2019) method and our proposed method can effectively solve this example. In addition, the decision results derived by the two methods are slightly different. The ranking order obtained by Gao et al.'s (2019) method is $A_1 \succ A_2 \succ A_3 \succ A_4$ and A_1 is the optimal alternative. Our method produces $A_1 \succ A_3 \succ A_2 \succ A_4$ and A_1 is also the best alternative. This result also indicates the effectiveness of our proposed method. In addition, both Gao et al.'s (2019) method and our proposed method have the following advantages, i.e., they can deal with DMs' unreasonable evaluation values and take the interrelationship among attributes into consideration. However, our proposed method is still more powerful than that proposed by Gao et al. (2019). This is because Gao et al.'s (2019) method is based on IFSs and our method is based on FFSs. As it is known, IFS should satisfy the condition that the sum of MD and NMD is less than or equal to one. Hence, FFS can describe larger information space than IFS and when using FFSs to describe DMs' evaluations information, DMs can have more freedom to express their evaluation opinions. Therefore, our method is more powerful and flexible than that developed by Gao et al. (2019).

6.4. Summary

Our proposed method provides a flexible and powerful manner for DMs to determine the ranking orders of alternatives in MAGDM problems. Advantages of our method are obvious, which can be summarized from three aspects. First, it is suitable to depict DMs' complex evaluation values in complicated realistic MAGDM problems. Our method is based FFSs and as analyzed above, FFSs have laxer constraint than IFSs and PFSs and hence FFSs can more comprehensively describe DMs' evaluation information. Second, our method is flexible to handle DMs' unreasonable evaluation values. In most MAGDM problems, due to many reasons (such as lack of expertise, time shortage, or prejudice), DMs may provide some extreme or unreasonable evaluation values, which, obviously negatively impact the final decision results. In other word, if such unreasonable evaluation values are not be felicitously handled, the final decision results are not reliable. As analyzed above, the second advantage of our method is that it can effectively cope with DMs' evaluation values. Finally, our method can take the interrelationship among attributes into consideration. In most real MAGDM problems, attributes are usually integrated and such kind of interrelationship among attributes should be considered when determining the final decision results. Hence, our method is powerful and suitable to be applied to handle realistic MAGDM problem.

7. Conclusions

This study dealt with MAGDM problems under FFSs. The FFS is a powerful tool to handle DMs' evaluation information in complex and uncertain decision-making situations. Hence, MAGDM method based on FFSs is a promising research filled.

This paper first proposed some new AOs for Fermatean fuzzy information, i.e., FFPA, FFPWA, FFPHM, and FFPWHM. In this study, properties of these operators were investigated in detail. These newly developed AOs have good performance in aggregating FFNs and hence, a novel MAGDM method based on these AOs was presented. The detailed steps of the new method were illustrated. Finally, we applied the proposed method in a real decision-making problems to demonstrate its rightness. Comparison analysis was provided to demonstrate the advantages of our method. Certainly, the limitations of this study should not be neglected. These study only considered a small group of DM, however, in some real and complex MAGDM problems, a large group of DMs are usually involved, which is called large scale group decision-making (LSGDM). Recently, LSGDM has received great attention (Gai et al., 2022; Zhou et al., 2022; Wang et al., 2022; Ma et al., 2022; Bai et al., 2022). Hence, in the future, we shall consider LSGDM methods under FFSs.

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