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Building a Mathematical Model to Determine the Optimal Production Quantity based on a Fuzzy Time Series: A Case Study

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Abstract. Production companies face many problems, including choosing the best suppliers of raw materials and the state of fluctuating demand for their products in the market. This study aims to determine the optimal quantities produced to achieve the highest possible profit return. Fuzzy time series were used to eliminate the state of uncertainty and instability in demand for products. Emphasis was placed on a real case study of one of the largest producers and suppliers of the liquid milk that manufactures various dairy products such as butter, ice cream, ghee and milk powder. Since the company is interested in its logistical plans concerning the supply of raw milk provided by four suppliers, to ensure that this raw material is sufficiently available, it prepares an assessment of each supplier to decide whether to continue to deal with it. A mathematical approach was used to determine the relative importance of each supplier to determine the quantities to be purchased from each of them and according to its evaluation. After studying the state of fluctuation in the fuzzy demand, it was processed by estimating the demands for each product, and building a mathematical model that achieves the highest net profit by determining the quantities purchased from raw milk, manufactured, sold, and stored quantities. The data that was adopted was based on the winter and summer seasons.

Keywords: Fuzzy time series, forecasting, supplier selection, Analytic Hierarchy Process (AHP), Lexicographic method, Linear programming.

1. Introduction

Most practical forecasting cases, such as financial time series, note the existence of difficult behaviours and patterns within the data, which makes their analysis and forecasting very complex. When looking for accurate prediction methods, data volume is another challenge. This scenario has stimulated, in recent years, the development of new methods to predict the alliance of predictive performance and low processing cost. One of these methods is the Fuzzy Time Series, which has gained more importance in recent years due to the direction of many studies that have proven its accuracy compared to other models.

The selection and evaluation of suppliers is an important part of optimal production management in most companies. The choice of the appropriate suppliers reduces the cost of purchasing raw materials, which makes the competitive activity in the market much better (Mehmet Sevkli, Selim Zaim, Ali Turkyılmaz and Metin Satır 2010). It is necessary to carefully consider the nature of the criteria when solving this kind of problem (H.T. Willis, R.C. Huston, and F. Pohlkamp 1993). For the decision-making process to be orderly, we create a pairwise comparison matrix; when performing these comparisons, we need a scale of numbers to determine the importance of one component concerning another for all components of the matrix. The goal of fuzzy time series is to form a forecasting period dependent on fuzzy limits out of the concept of probability (Q. Song and B. S. Chissom 1993), (S. M. Chen 1996). (Hajar Shikhaliyevaa 2016) presents a new approach to defuzzification of results of fuzzy time series dependent on applying the fuzzy set point-appreciation method.

This article is devoted to a real case study of one of the largest producers and suppliers of liquid milk, which manufactures various dairy products (butter, ice cream, ghee and milk powder). The company is interested in developing its production plans to meet the unstable demand for its products with the supply of raw milk provided by four suppliers. To ensure that this raw material is sufficiently available, it prepares an assessment of each supplier to decide whether to continue to deal with it. A mathematical approach (Analytic Hierarchy Process) was used to determine the relative importance of each resource to determine the quantities to be purchased from each of them and rate it. After studying the fluctuation in the fuzzy demand, it was treated by estimating the demand for each product and building a mathematical model that achieves the highest net profit by determining the quantities purchased from raw milk and the quantities manufactured, sold and stored. The adopted data was based on the two seasons (winter and summer) from eleven years ago.

2. Literature Review

Data analysts seek to forecast market demand based on their professional knowledge. Higher accuracy is the most important because more profit will be made if accurate predictions are given. For more than two decades, various fuzzy time series models have also been applied to solve problems in many areas, such as financial forecasting (Faff, Brooks, Kee, 2002; Huarng , Yu, 2005; Shin , Sohn, 2004; Wang, 2002; Yu, 2004), university enrollment forecasting (Chen, 1996; Song , Chissom, 1993, 1994), Through the link between suppliers and consumers, manufacturers can build and practice a ranking program according to the importance of suppliers, which directly affects the performance of the company (R. K. Malviya and R. Kant, 2015, K. Zimmer, M. Frohling, and F. Schultmann, 2016).

Suppliers, as partners in the primary supply chain, play an important role in achieving company goals. Thus, selecting the most qualified supplier in the supply chain is a vital strategic decision to maintain the competitive position of the company (N. Jain and A. R. Singh 2020). Supplier selection can be considered as a complex decision-making problem with the aim of ensuring better performance from enterprise suppliers (Tasnia Hassan Nazifa, K.K Ramachandran 2019).

3. Research Method

3.1. Determine if the selection criteria

Companies devote attention to their logistics plans when it comes to providing raw materials and services. The suppliers of these materials affect the performance of any company through its profits and levels of service. For companies to ensure that these materials are adequately available, they prepare an assessment for each supplier to make decisions about whether or not to continue dealing with them.

The basic steps of the supplier selection process are summarised in three steps:

1- Identifying the available suppliers.

2- Determining the criteria for comparison between them.

3- Select the supplier.

The search for suppliers is done when competition enters a new market or when the company desires to renew the group of existing suppliers. Definition of the criteria for differentiating suppliers with their relative weight is the most important step in decision-making.

There must be harmony between the selection of criteria of comparison among suppliers and the company's objectives; therefore, it is not correct to specify a list that applies to all the selection criteria according to the different objectives of the companies. Because there are many differentiation criteria, the decision-maker must put relative weight on each criterion. To find the weights of criteria, the following two steps can be taken (Lawrence V. Snyder, Ya-xiang Yuan 2013):

- Let c be the number of comparison criteria between suppliers, and Z is a matrix by $c \times c$, and it has the elements e_{ij} $, i = 1,2, \ldots, c,\ j = 1,2, \ldots, c$ $, i \neq j$, which is represented to the relative importance of criterion i to criterion j, as shown in Table (1), and $e_{ij} = \frac{1}{e_{ji}}$, and $e_{ii} = 1$.

  For all criteria, calculate the value of τ_i , which refer to the geometric mean of the elements of the i^{th} row of matrix Z.

$$\tau_i = \sqrt[c]{\prod_j^c e_{ij}} \quad , i = 1,2, \ldots, c \tag{1}$$

- Each criterion has a weight, founded by the formula below:

$$\omega_i = \frac{\tau_i}{\sum_i^c \tau_i} \quad , i = 1,2, \ldots, c \tag{2}$$

Table 1: the values of pairwise comparison (scale of importance) According to (Thomas L. Saaty , Luis G. Vargas 2013)

Degree of importance	Definition
1	Similar of importance
2	Weakly
3	Mild
4	Mild plus
5	Strong importance
6	Strong plus
7	Very strong
8	Very ,very strong
9	Extreme importance

3.2. Specify the suppliers

There are many different mathematical methods for selecting suppliers, but we will depend on the weighted scoring method. Let Ψ be the potential suppliers, ω_i represents the criterion's weight to the rest of the criteria, whose value is between zero and one, obviously $\sum_i^c \omega_i = 1$. Let υ_{si} is the score of the supplier (s) according to criterion(i), which is to take the value (0 to 10). The total score of the supplier (s) where s belong toΨ, is:

$$\mathrm{T}\upsilon_s = \sum_i^c \upsilon_{si} \omega_i \tag{3}$$

It is possible to highlight the supplier with the highest score among peers; this means$(s^{opt}) = maximum\ value\ of\ \mathrm{T}\upsilon_s$. One of the simplest models for formulating the problem of selecting suppliers as a multi-objective linear programming model is as follows:

The first objective is to minimise the overall purchasing cost,

$$\min z = \sum_{\forall s \in \Psi} uc_s \, x_s \quad , \tag{4}$$

The second objective is to maximise the total score of the chosen suppliers,

$$\max z = \sum_{\forall s \in \Psi} \mathrm{T}\upsilon_s / \arg \max\{\mathrm{T}\upsilon_s\} x_s \tag{5}$$

The sum of the quantities purchased from the supplier (S) equals the total demand,

$$\sum_{\forall s \in \Psi} x_s = Đ, \tag{6}$$

The sum of the quantities purchased from the supplier (s) does not exceed the maximum ability of supplier (s),

$$\sum_{\forall s \in \Psi} x_s \leq cap_s, \tag{7}$$

The non-negative constraint is $x_s \geq 0 \;, s \in \Psi$.

To solve the multi-objective model problem, *the Lexicographic method* (Barraq Subhi Kaml, Mohamed Saad Ibrahim 2018) is used; this method depends mainly on arranging the objective functions according to their importance according to what the decision-maker sees so that the preferred solution that we get is the one that maximises the other objective functions so that there is no exchange between objectives for different levels of precedence, and it is expressed mathematically:

$$\min f_g(x)$$

subject to

$$f_k(x) \leq f_k^{opt}, k = 1,2,\ldots,g-1 \tag{8}$$

3.3. Fuzzy time series model

To treat the case of fluctuation in demand, the steps of the time-series algorithm will be discussed and can be summarised as follows (Hajar Shikhaliyeva,Tahir Mehdiyev 2016):

Step1: Determining the lower and upper limits of the demand historical data series by the formula:

$$L = [Ld_{min} - \bar{p}, Ld_{max} + \overline{\overline{p}}] \tag{9}$$

Where:

L: The lower and upper limits of the demand historical data series.

Ld_{min}: Minimum value of historical data.

Ld_{max}: Maximum value of historical data.
$\bar{p},\overline{\overline{p}}$: Positive numbers determined by the decision-maker according to the nature of the data.

Step2: Partition the data into equal-length intervals$(I_1, I_2, \dots, I_c)$; then, the fuzzy groups were determined based on the earlier periods $(fb_1, fb_2, \dots, fb_c)$.
Step3: Fuzzification of historical data series and fuzzy groups, fuzzified each value of the data according to the fuzzy group to which it belongs.

Step4: Determine the fuzzy relationships in fuzzified data, if the time variable N_{t-1} is fuzzify up as fb_c and N_t is up as fb_h that means correlated fb_c to fb_h , expresses it as $fb_c \to fb_h$.

Step5: Calculate the midpoints of intervals time I; for example, if $fb_j \to fb_c$ and the degree of membership of fb_c occurs in the interval I_c , then the midpoints are equal to the sum of two terms of interval I_c divided by 2.

Step6: After determining the midpoints, the fuzzy orders are processed using the median method, where

$$x_{median} = \frac{med_1 + med_2 + \cdots + med_c}{c} \tag{10}$$

Step7: forecasting of demand by using the trend line method,

$$\hat{y} = \alpha + \beta x \tag{11}$$

4. Case study (Data description)

ABC Company is one of the largest companies in producing and supplying liquid milk and various dairy products such as butter, ice cream, ghee, and milk powder (Katta G. Murty 2015). The company has four raw milk suppliers, and they are provided at the prices indicated in Table (5). Its average daily purchase of raw milk from its suppliers is 7 million litres. Its products can be categorised into two categories: high-yield products (butter, ghee, and ice cream) and low-yield products (milk powder). Milk powder is non-perishable and does not require refrigeration. In addition, milk powder can be reconstituted into liquid milk used as basic raw materials. The planning horizon is annually divided into two seasons, winter and summer—about every half year.

This company competes in the market for high-return products. One of its strategies is not to lose market share for any of the low-return products. This is achieved through its policy of meeting the minimum customer demand in each product category. For this, they have established their retail outlets in many locations, although it has not been noticed that it is a profitable proposition in many places. As a first stage, the problem is that there is a difference between the suppliers of raw milk, so the company, based on the data listed in Table (2), gives preference to one supplier over another.

Decisions to be made specifying each of the winter and summer seasons include:

1. Purchasing price and different quantities of each supplier of raw milk.
2. The sales volume of each output product.
3. Production level of each product.

To help make these decisions, data is provided: purchase data, production and sales volumes, and production capabilities, seasonal basis. To evaluate a company's raw milk suppliers, the following four criteria of preference (numbered 1 through 4) are used: quality of raw milk, cost, a commitment of delivery time, and geographic location. To determine the weight of each criterion, the purchasing office manager has adopted the standard scale of values shown in Table (1) to get the judgments shown in Table (2) from the pairwise comparison. Thus, the matrix Z with dimension 4 x 4 is:

$$Z = \begin{bmatrix} 1 & 1 & 7 & 9 \\ 1 & 1 & 5 & 3 \\ 1/7 & 1/5 & 1 & 9 \\ 1/9 & 1/3 & 1/9 & 1 \end{bmatrix}$$

Table 2: An evaluation of the four suppliers according to each criterion (the degree of evaluation from 0 to 10)

criterion	**Supplier A**	**Supplier B**	**Supplier C**	**Supplier D**
Quality of milk	7	8	7	9
Purchasing cost	8	6	8	7
A commitment to delivery time	6	7	9	8
Geographical location	9	5	7	6

Table 3: The Demand for products

Year	Milk Powder (million kg)		Ghee (million kg)		Butter (million kg)		Ice-cream (million kg)	
	winter	Summer	Winter	Summer	Winter	Summer	Winter	Summer
1	96.75	64.5	1.08	0.72	4.85	3.23	4.26	5.43
2	101.91	62.46	1.28	0.78	5.41	3.32	4.86	6.77
3	106.11	76.84	1.44	1.18	5.14	4.2	7.43	7.58
4	103.63	66.26	1.78	1.29	6.45	4.67	7.69	8.96
5	127.18	74.7	2.35	1.38	8.06	4.73	7.46	10.86
6	131.57	87.71	2.78	1.85	8.59	5.73	9.51	12.11
7	131.77	91.57	3.52	2.43	9.55	6.64	11.6	14.12
8	140.15	101.49	4.54	3.29	10.79	7.82	14.5	16.88
9	162.81	91.58	6.51	3.66	13.94	7.84	14.42	21.99
10	172.24	92.75	8.53	4.60	16.98	9.15	17.1	27.32
11	157.28	92.37	10.84	6.36	20.41	11.99	20.55	29.95

Table (4) shows the company's production capacity with production cost and inventory holding cost from season to season for each of the final products during the planning year. The Table also provides information on the required need for each litre of raw milk to produce one unit of this product.

Table 4: Production capacity, production/holding cost and raw milk requirements for finished products

Products	The ability to produce (million kg)	Production cost ($/kg)	Holding cost ($/kg)	Requirements of each product of raw milk (litre)
Milk powder	100	0.16	0.02	6.4
Ghee	25	0.43	0.05	14.8
Butter	27.5	0.35	0.04	10.5
Ice-cream	27.5	0.23	0.03	3.75

Table 5: purchasing cost of raw milk, the sale price of products and the capacity of each supplier

suppliers	Capacity of supplier (daily/million litre)	purchasing cost ($/Liter)		products	sale price ($/kg)	
		Winter	Summer		Winter	Summer
A	2.25	0.10	0.13	Milk powder	2.75	2.75
B	4.15	0.12	0.14	Ghee	4.70	3.60
C	3.75	0.09	0.13	Butter	4.25	3.25
D	4.25	0.11	0.15	Ice-cream	1.50	1.75

The goal of ABC Company is to maximise the net profit for each season [aggregate sales revenue – (purchase cost of raw milk + production cost + inventory holding cost)].

5. Building a mathematical model of the problem

5.1. Mathematical model to specify the best suppliers

Before starting to build the mathematical model, it is required to determine the best raw milk supplier, as follows:

By using equation (1), we obtain:

$$\tau_1 = 2.82, \tau_2 = 1.97, \tau_3 = 0.71, \tau_4 = 0.25$$

Each criterion has a weight, founded by the formula (2), we get:

$$\omega_1 = \frac{2.82}{5.75} = 0.49 \;, \omega_2 = \frac{1.97}{5.75} = 0.34, \omega_3 = \frac{0.71}{5.75} = 0.12 \;, \omega_4 = \frac{0.25}{5.75} = 0.04$$

The criterion with the high score in weight (49%) is the quality of raw milk, followed by the purchasing cost of raw milk with a weight (34%).

The next step is to determine which supplier has the highest score using the equation (3):

criterion	Weights of criteria	Supplier A	Supplier B	Supplier C	Supplier D
Quality of raw milk	0.49	7	8	7	9
purchasing cost	0.34	8	6	8	7
A commitment to delivery time	0.12	6	7	9	8
Geographical location	0.04	9	5	7	6
Score		7.23	7	7.51	7.99

Then, we use equations (4-7) as a multi-objective linear programming model:

$$\min z = 0.10\, x_{AW} + 0.13 x_{AS} + 0.12 x_{BW} + 0.14 x_{BS} + 0.97 x_{CW} + 0.13 x_{CS} + .11 x_{DW} + 0.15 x_{DS}$$

$$\max z = 7.23/7.99\, x_{AW} + 7.23/7.99 x_{AS} + 7/7.99 x_{BW} + 7/7.99 x_{BS} + 7.51/7.99 x_{CW} + 7.51/7.99 x_{CS} + 7.99/7.99\;\; x_{DW} + 7.99/7.99 x_{DS} \qquad (12)$$

Subject to

$$x_{AW} + x_{AS} + x_{BW} + x_{BS} + x_{CW} + x_{CS} + x_{DW} + x_{DS} = 7 * 365$$
$$x_{AW} + x_{AS} \leq 2.25 * 365, x_{BW} + x_{BS} \leq 4.15 * 365,$$
$$x_{CW} + x_{CS} \leq 3.75 * 365, \; x_{DW} + x_{DS} \leq 4.25 * 365 \quad (13)$$

We use the traditional lexicographic method, using system of equations (8) by making the obtained objective function constraints within the constraints of the problem and equating them with their obtained value as follows:

$$\min z = 0.10\, x_{AW} + 0.13x_{AS} + 0.12x_{BW} + 0.14x_{BS} + 0.97x_{CW} + 0.13x_{CS} + 0.11x_{DW} + 0.15x_{DS} \quad (14)$$

Subject to

$$0.9x_{AW} + 0.9x_{AS} + 0.87x_{BW} + 0.87x_{BS} + 0.94x_{CW} + 0.94x_{CS} + \; x_{DW} + x_{DS} \; \leq 6190$$

$$x_{AW} + x_{AS} + x_{BW} + x_{BS} + x_{CW} + x_{CS} + x_{DW} + x_{DS} = 2555$$
$$x_{AW} + x_{AS} \leq 851.25, \qquad x_{BW} + x_{BS} \leq 1514.75$$
$$x_{CW} + x_{CS} \leq 1368.75, \quad x_{DW} + x_{DS} \leq 1551.25 \quad (15)$$

All variables ≥ 0

The optimal solution of quantities to be purchased from each supplier, by using WINQSB (Yih-Long C. 2002) is:

$$x_{AW} = 172, x_{AS} = 71 \;\;, x_{BW} = 250 \;\;, x_{BS} = 78,$$
$$x_{CW} = 535, x_{CS} = 408 \;\;, \quad x_{DW} = 595, x_{DS} = 446$$

5.2. Forecasting of fuzzy demand

To solve the case of fluctuation in demand for milk powder and the other products in the same way, the steps of the time-series algorithm from step1 to step3 in paragraph (2) will be applied as follows:

$$L = [Ld_{min} - \bar{p}, Ld_{max} + \overline{\overline{p}}],$$
$$L = [96.75 - 1.75, 172.24 + 10.76], \; L = [95,183]$$

Table 6: illustrate intervals of data and fuzzy groups

Year No.	Historical data (winter) (million kg)	Intervals	Fuzzy groups	Historical data (summer) (million kg)	Intervals	Fuzzy groups
1	96.75	[95,103]	F_{b1}	64.5	[62,66]	F_{b1}
2	101.91	[103,111]	F_{b1}	62.46	[66,70]	F_{b1}
3	106.11	[111,119]	F_{b2}	76.84	[70,74]	F_{b4}
4	103.63	[119,127]	F_{b2}	66.26	[74,78]	F_{b2}
5	127.18	[127,135]	F_{b5}	74.7	[78,82]	F_{b4}
6	131.57	[135,143]	F_{b5}	87.71	[82,86]	F_{b7}
7	131.77	[143,151]	F_{b5}	91.57	[86,90]	F_{b8}
8	140.15	[151,159]	F_{b6}	101.49	[90,94]	F_{b10}
9	162.81	[159,167]	F_{b9}	91.58	[94,98]	F_{b8}
10	172.24	[167,175]	F_{b10}	92.75	[98,102]	F_{b8}
11	157.28	[175,183]	F_{b8}	92.37	[102,106]	F_{b8}

Then determine the relationships between fuzzy groups as in table (7).

Table 7: the relationships among fuzzy groups

winter				summer			
G1	$F_{b1} \to F_{b1}$ $F_{b1} \to F_{b2}$	G4	$F_{b6} \to F_{b9}$	G1	$F_{b1} \to F_{b1}$ $F_{b1} \to F_{b4}$	G4	$F_{b7} \to F_{b8}$
G2	$F_{b2} \to F_{b2}$ $F_{b2} \to F_{b5}$	G5	$F_{b9} \to F_{b10}$	G2	$F_{b4} \to F_{b2}$ $F_{b4} \to F_{b7}$	G5	$F_{b8} \to F_{b8}$ $F_{b8} \to F_{b10}$
G3	$F_{b5} \to F_{b5}$ $F_{b5} \to F_{b5}$ $F_{b5} \to F_{b6}$	G6	$F_{b10} \to F_{b8}$	G3	$F_{b2} \to F_{b4}$		

After determining the midpoints, the fuzzy data are processed using the equation (10)

Table 8: Treatment of the expected fuzzy output

Year No.	**Mid-points of Intervals** (winter)	**Crisp demand** (winter)	**Mid-points of Intervals** (summer)	**Crisp demand** (summer)
1	{99,107}	103	{64,76}	70
2	{99,107}	103	{64,76}	70
3	{107,131}	119	{68,88}	78
4	{107,131}	119	{76}	76
5	{131,131,139}	133.67	{68,88}	78
6	{131,131,139}	133.67	{92}	92
7	{131,131,139}	133.67	{92,100}	96

8	{163}	163	{92}	92
9	{171}	171	{92,100}	96
10	{155}	155	{92,100}	96
11	{171}	171	{92,100}	96

After the demand undergoes fuzzy and is processed, the trend line method as in equation (11) is used to forecast the demand for the next year.

Table 9: illustrate Expected order quantities for ABC products for the winter and summer seasons

products	**Forecasting demand**	
	Winter(million kg)	**Summer(million kg)**
Milk Powder	180	102
Ghee	11.74	5.66
Butter	20	11.3
Ice- cream	20.43	31.6

According to determine the quantities of raw milk to be purchased from each supplier, address the fluctuation in demand for the company's final products, and estimate the demand for the next year, it is now possible to build a mathematical model of the problem to determine the maximum net profit that the company can achieve, as below:

Indexes

s: Numbers of suppliers of raw milk, $s = 1,2,3,4$.
$k: final\ products, k = 1,2,3,4$.
$j: represent\ season\ winter\ and\ summer, j = 1,2$.

Data components

$\theta^{kj}: The\ selling\ price\ of\ the\ product\ k\ in\ season\ j, k = 1,2,3,4,$
$j = 1,2.$
$\rho^{ij}: purchase\ cost\ of\ raw\ milk\ from\ supplier\ i\ in\ season\ j, i =$
$1,2,3,4,$
$j = 1,2.$
$\delta^{kj}: \ The\ cost\ of\ producing\ one\ unit\ in\ kg\ of\ the\ product\ is\ k$
$1,2,3,4,$
$j = 1,2.$
$\beta^{kj}: the\ holding\ cost\ per\ unit\ of\ product\ k\ in\ season\ j, k =$
$1,2,3,4,$
$j = 1,2.$

μ_{kj}: *Quantities of demand for the product k in season* $j, k = 1,2,3,4,$ $j = 1,2.$

η_{kj}: *The ability to produce product k in season* $j, k = 1,2,3,4,$ $j = 1,2.$

λ_{kj}: *The required quantity of raw milk in litres to produce one unit of the* *season* $j,\ k = 1,2,3,4,\ j = 1,2.$

$\mathcal{H}_j$: *Quantity of raw milk to be purchased from suppliers in the season* $j, j = 1,2.$

Ω_j: *The company's need of raw milk in the season* $j, j = 1,2.$

Decisions variables

x_{kj}: *Quantity of product k that sold in season* $j, k = 1,2,3,4, j = 1,2.$

x_{ij}: *Quantity of raw milk that purchase from supplier i in seas* $1,2,3,4,$ $j = 1,2.$

w_{kj}: *Quantity of product k that manufactured in season* $j, k = 1,2,3,4,$ $j = 1,2.$

l_{kj}: *Quantity of product k that stock in season* $j, k = 1,2,3,4, j = 1,2.$

$$\max z = \sum_{k=1}^{4}\sum_{j=1}^{2}\theta^{kj}x_{kj} - \left[\sum_{s=1}^{4}\sum_{j=1}^{2}\rho^{ij}\, x_{ij} + \sum_{k=1}^{4}\sum_{j=1}^{2}\delta^{kj}w_{kj} + \sum_{k=1}^{4}\sum_{j=1}^{2}\beta^{kj}l_{kj}\right] \tag{16}$$

Subject to

$$\sum_{k=1}^{4}\sum_{j=1}^{2}x_{kj} \geq \mu_{kj}, \sum_{k=1}^{4}\sum_{j=1}^{2}x_{kj} \leq \eta_{kj}, \sum_{k=1}^{4}\sum_{j=1}^{2}\lambda_{kj}x_{kj} \leq \mathcal{H}_j, \sum_{s=1}^{4}\sum_{j=1}^{2}x_{ij} \leq \Omega_j$$

$$x_{kj}, x_{ij}, w_{kj}, l_{kj} \geq 0 \tag{17}$$

6. Results and Discussion

From Table (10), we note that the optimal production plan requires ABC Company to make fully use of its production capacity in the winter for three production products - milk powder, butter, and ice cream. In contrast, the production capacity of ghee remains unexploited.

The milk powder, butter, and ice cream sales volume is lower than the demand for products in the summer season. It is also interesting to note that the ABC

Company ensures milk supply in the summer season by fully meeting the minimum target sales of milk powder in the winter season and limiting the excess production of milk powder that can be reconstituted into liquid milk in the summer season as desired.

Table10: The Results and the optimal solution.

Quantity allocated of the raw milk to be purchased (million/litre)		**product**	**Quantity of product that is manufactured (million/kg)**		**Quantity of products that sold (million/kg)**		**Quantity of product that stock (million/kg)**	
winter	**Summer**		**winter**	**summer**	**winter**	**summer**	**winter**	**summer**
1552	1003	**Milk Powder**	195	87	180	102	15	0
		Ghee	13	4.4	11.74	5.66	1.26	0
		Butter	23.3	8	20	11.3	3.3	0
		Ice-cream	25	27.03	20.43	31.6	4.57	0

Table (11) shows the total revenues generated from selling the four products, the costs of purchasing raw milk, production costs, and the holding cost.

Table 11: Finding the net profit

Revenues($/million)								**Cost of Raw milk**		**production cost**		**Holding cost**	
Milk Powder		**Ghee**		**Butter**		**Ice- cream**							
w	S	w	S	W	s	w	s	w	s	w	s	w	s
495	229.5	55.2	20.4	85	36.8	30.6	55.3	160.8	152.9	50.7	24.8	0.63	0
Total	**1007.8**							**300.9**		**75.5**		**0.63**	

The net profit = [Aggregate sales revenue – (purchase cost of raw milk + production cost + inventory holding cost)]

The net profit = 1007.8-(300.9+75.5+0.63)

The net profit =630.77 $/million

7. Conclusion

The supplier selection rule is critical to the success of a production company. Accordingly, the buy of raw materials from exterior suppliers may compose a large ratio of a commodity’s costs; suppliers should be accurately compared with each other to assign their relative importance of toughness and weaknesses. The application of fuzzy logic is an effective way to eliminate uncertainties in the industrial environment, which lacks a certain state. The use of time series also contributed greatly to identifying and addressing fluctuations in demand.

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