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On the determination of optimal stock levels in a discrete demand context based on fill rate criterion

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Abstract. This paper focuses on the estimation of the base stock level, S, based on the service approach when the fill rate criterion is used. The fill rate is defined as the fraction of demand that is directly satisfied with the on-hand stock. Although this definition is simple, the fill rate has been traditionally calculated through the known as traditional approximation, i.e. computing the units short instead of computing directly the satisfied demand. As a consequence, the application of the traditional approximation to determine the optimal S can lead to an unnecessary increase of the stock under some circumstances. This paper assesses when approximate methods based on the traditional approach provide an accurate calculation of the S when a target fill rate is given, the inventory is managed by the periodic review policy and the demand follows any discrete distribution. To this aim, we carry out a wide experiment whose results are analyzed by using Classification and Regression Trees technique. As a result it is possible to identify regions of similar performance of the approximate methods and represent them in a novel space of representation delimited by the coefficient of variation of demand and the average demand over the revision period plus the lead time. Furthermore, risks of using any of the approximate methods to compute S in every region are evaluated. This analysis is summarized in a reference framework that can be used by managers in the decision-making process of determining the best method to compute base stock levels.

Keywords: base stock; fill rate; discrete demand, optimal inventory policies

1. Introduction

One of the traditional problems in inventory management consists of how to compute accurately the base stock level in a periodic review inventory system. It can be used the cost approach trying to find the optimal policy that minimizes the total costs of the system. The difficulty of this approach is the computation of the penalty cost. In practice it is hard to quantify the cost incurred by a stockout (Larsen and Thorstenson, 2008). In this sense, Liberopoulos (2010) distinguishes two types of cost in a stockout situation: the direct cost, which can be easily calculated as the demand that is lost or backordered, and the indirect cost, which is actually the hard part to evaluate. The indirect cost is related to the loss of customers goodwill and image of the company which may lead to a decrease of future demand. Furthermore, from a managerial perspective it is necessary to take into account that costs may vary over time due to the inflation or process improvements (Buzacott and Shanthikumar, 1993; Silver, 2008). Another approach to find the optimal parameters of the inventory system focuses on setting a target service level and computing the base stock that guarantees its achievement. As Bijvank and Vis (2012) point out this service approach is useful when a service level restriction is imposed by the supply chain and it is easier to define a target service level than all the costs. To implement the service approach first of all it is required to select the service measure. In this sense the fill rate has been widely used. This metric is commonly defined as the fraction of demand that is immediately satisfied from shelf. However, this simple-looking definition hides a lot of technical details and nuances that are often overlooked in the literature. In fact, the fill rate has been simplified through the traditional approximation which computes the fill rate in terms of units short, i.e. as the complement of the quotient between the expected unfulfilled demand and the total expected demand. As a consequence, the traditional approximation tends to underestimate the fill rate and therefore, when it is used to determine the optimal inventory policy, it gives a stock level greater than the exact one representing an increase in holding costs (Guijarro et al, 2012).

The aim of this paper is to assess under which conditions approximate expressions based on the traditional approximation can provide an accurate enough estimation of the optimal base stock level in a discrete context. To accomplish this objective, we carry out a large experiment with the aim of: (i) finding the simplest and most accurate method to compute the base stock level; (ii) assessing errors that arise from using an approximate method based on the traditional approximation instead of the exact one and (iii) developing a reference framework which is built by using Classification and Regression Trees (CART), a statistical exploratory technique which allows identifying when the approximate methods perform well, i.e. compute exactly the base stock level or when the exact method is required.

This reference framework can be used by managers to decide the most efficient method to compute the base stock once the target service level is defined. As Silver (2008) affirms "an understandable decision rule that improves somewhat on current conditions is almost certainly better than the optimal solution that is neither understood nor accepted by management". Thus, the main contribution of this work is the proposal of an easily understood and applicable reference framework to design optimal inventory policies in a discrete demand context. This reference framework integrates the characteristics of the inventory system and the demand pattern of the item. Furthermore, risks which arise from using an approximate method in any context are also assessed.

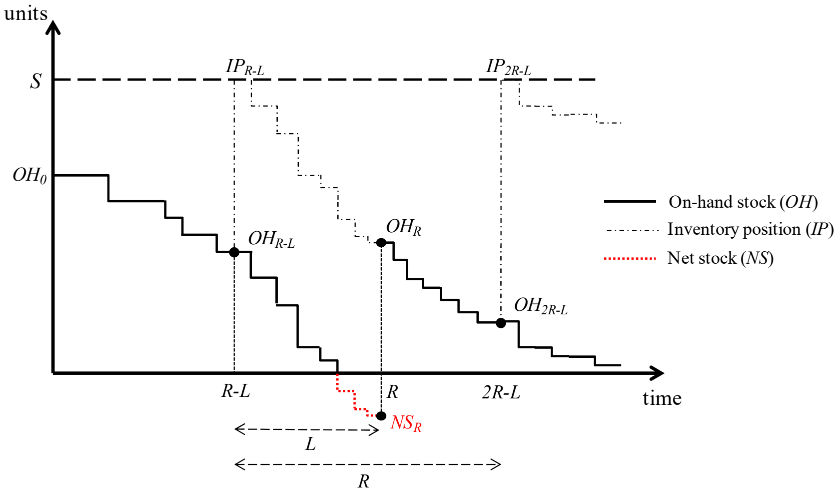
The remainder of the paper is organized as follows. Section II presents the notation and assumptions followed in this paper. Section III dedicates to a literature review about different interpretations of the fill rate definition and presents the available methods to compute the fill rate in a periodic review system. The numerical experiment and the analysis carried out are explained in Section IV. Finally, Section V sums up the main conclusions of this work and highlights its practical applications.

2. Basic Notation and Assumptions

This paper is concerned with the traditional periodic review, order up to level (R, S) system that launches a replenishment order every R units of time of sufficient magnitude to raise the inventory position to the base stock, S. The replenishment order is received L time periods after being launched. Figure 1 shows an example of the inventory evolution in a (R, S) inventory system.

We focus on a single item and consider that the constant review interval has been previously determined. In the rest we assume that: (i) time is discrete and organized in a numerable and infinite succession of equally spaced instants; (ii) the lead time is constant and known; (iii) unfilled demand is complete backlogged to the next period and served as soon as the next replenishment order arrives; (iv) the replenishment order is added to the inventory at the end of the period in which it is received, hence these products are available for the next period; (v) demand during a period is fulfilled with the on-hand stock at the beginning of the period; and (vi) the demand process is considered stationary, i.i.d., and defined by any discrete function.

Figure 1. On-hand stock, inventory position and net stock evolution in a (R, S) periodic review



The notation used in Figure 1 and in the rest of the paper is as follows:

S = base stock (units),

L = lead time for the replenishment order (time units),

R = review period and replenishment cycle corresponding to the time between two consecutive deliveries (time units),

OHt = on-hand stock in time t from the first reception (units),

NSt = net stock in time t from the first reception (units),

IPt = inventory position in time t from the first reception (units),

Dt = accumulated demand during t consecutive periods (units),

ft(·) = probability density function of demand in t,

Ft(·) = cumulative distribution function of demand during t periods,

E(X) = expected value of a random variable X.

3. The Fill Rate in Periodic Review Systems

The fill rate definition

The fill rate is one of the service measures most used in practice since it considers not only the possibility that the system is out of stock, but also the size of the unfulfilled demand when it occurs (Tempelmeier, 2007). This metric is commonly defined as the fraction of demand that is immediately satisfied from shelf and can be expressed as (Guijarro et al, 2012):

$$FR = E\left(\frac{fulfilled\ demand}{total\ demand} \mid positive\ demand\ during\ cycle \right) \quad (1)$$

Although its definition is simple, we find different interpretations of it in the literature and consequently different expression to compute the fill rate. In fact only few works suggest computing the fill rate following expression (1), i.e. estimating directly the expected fulfilled demand per replenishment cycle. However, we find several works that propose the estimation of the fill rate through the known as traditional approximation which computes the numbers of units not satisfied, instead of computing directly the fulfilled demand, as:

$$FR_{Approx} = 1 - \frac{E\left(unfulfilled\ demand\right)}{E\left(total\ demand\right)} \quad (2)$$

or using its complement:

$$FR_{Approx} = \frac{E\left(fulfilled\ demand\right)}{E\left(total\ demand\right)} \quad (3)$$

Nevertheless, the traditional approximation tends to underestimate the fill rate (Guijarro et al, 2012; Babiloni et al, 2012). An important consequence of the underestimation behaviour is found when the traditional approximation is used to determine the base stock since underestimating the fill rate supposes overestimating the base stock and therefore increasing holding costs of the system. Thus, from a practical point of view it seems useful to know under which conditions approximate methods based on (2) and (3) can provide an accurate enough approximation to the optimal stock level for periodic review systems. This paper answers this question.

Expressions of the fill rate in a periodic review (R, S) inventory system

As previous section pointed out, only a handful of papers proposes exact expressions to compute the fill rate following (1). To the best of our knowledge, in a periodic review (R, S) system, only Babiloni et al (2012) derive and exact calculation of the fill rate based explicitly on its definition, i.e., directly estimating the fraction of the fulfilled demand per replenishment cycle instead of the expected shortage:

$$FR_{Exact} = \sum_{NS_0=1}^{S} f_L\left(S - NS_0\right) \cdot \left\{ \frac{F_R\left(NS_0\right) - F_R\left(0\right)}{1 - F_R\left(0\right)} + \sum_{D_R = NS_0+1}^{\infty} \frac{NS_0}{D_R} \cdot \frac{f_R\left(D_R\right)}{1 - F_R\left(0\right)} \right\} \quad (4)$$

Note that this expression (named Exact henceforth) can be applied for any discrete demand distribution in a backordering case.

However, we find quite a number of works suggesting methods to estimate the fill rate following expressions (2) or (3) in different contexts. In both cases, the expected total demand can be straightforwardly computed so the difference resides in how to compute the expected unfulfilled demand (in expression (1)) or the expected fulfilled demand (in expression (2)). Restricting for the periodic review (R, S) system, we find the following approximations:

- the traditional approach (*Trad* in the rest of the paper) that is presented in classical Operation Management texts (as Silver (1998) among others) and widely used in practical environments due to its simplicity.
- the fill rate expression based on the expected shortage per replenishment cycle suggested by (Hadley and Whitin, 1963) (henceforth, *H&W*) which is used as the numerator of (2) by several authors such as (DE Kok, 1990; Heijden and DE Kok, 1998; Silver and Bischak, 2011).
- the method presented in (Silver, 1970) and reformulated by (Johnson, 1995) for periodic review policy (*R*, *S*) when demand is normally distributed (*Silver* in the rest).
- the approximation derived by (Johnson, 1995) to compute the fill rate when the review cycle is one period and demand follows a normal pattern (henceforth, *Johnson*). Note that μ represents the mean demand during a cycle in the expression of Table 1.

- the approach proposed in Teunter(2009) (*Teunter* from now on) to calculate the fill rate for any continuous demand process in a single-stage general periodic review inventory system.
- the fill rate method derived by (Babiloni et al, 2012) to compute expression (1) in a periodic review system when demand follows any discrete demand distribution (*Babiloni* in the rest).

The majority of these approximations assumes continuous demand pattern, most often with the normal distribution. Discrete demands are only considered by Hadley and Whitin (1963), who restrict their analysis to Poisson distribution, and by (Babiloni et al, 2012) who assume any discrete demand function. Since this paper focuses on discrete demand context, we consider the discretization of the above expressions following Guijarro et al (2012). Table 1 summarizes the discrete formulation of the approximations considered in this work.

Table 1. Approximate fill rate expressions in a (R, S) inventory system and discrete demand context

Author/s	**Expression**
Traditional	$FR_{Trad} = 1 - \frac{\sum_{D_{R+L}=S}^{\infty} (D_{R+L} - S) \cdot f_{R+L}(D_{R+L})}{\sum_{D_R=1}^{\infty} D_R \cdot f_R(D_R)}$
Hadley&Whitin (1963)	$FR_{H\&W} = 1 - \frac{\sum_{D_{R+L}=S}^{\infty} (D_{R+L} - S) \cdot f_{R+L}(D_{R+L}) - \sum_{D_L=S}^{\infty} (D_L - S) \cdot f_L(D_L)}{\sum_{D_R=1}^{\infty} D_R \cdot f_R(D_R)}$
Silver (1970)	$FR_{Silver} = \frac{\sum_{D_{R+L}=-\infty}^{S} D_R \cdot f_{R+L}(D_{R+L}) + \sum_{D_{R+L}=S}^{S+D_R} (S + D_R - D_{R+L}) \cdot f_{R+L}(D_{R+L})}{\sum_{D_R=1}^{\infty} D_R \cdot f_R(D_R)}$

Johnson et al. (1985)	$FR_{Johnson} = \frac{\sum_{D_{R+L-1}=-\infty}^{S} \sum_{D_I=S-i}^{\infty} (D_I + D_{R+L-1} - S) \cdot f_I(D_I) \cdot f_{R+L-1}(D_{R+L-1}) + \mu \cdot (1 - F_{R+L-1}(S}{\sum_{D_R=1}^{\infty} D_R \cdot f_R(D_R)}$
Teunter (2009)	$FR_{Teunter} = 1 - \frac{\sum_{D_L=0}^{S} (S - D_L) f_L(D_L) - \sum_{D_{R+L}=0}^{S} (S - D_{R+L}) f_{R+L}(D_{R+L})}{\sum_{D_R=1}^{\infty} D_R \cdot f_R(D_R)}$
Babiloni et al. (2012)	$FR_{Babiloni} = 1 - \frac{\sum_{NS_0=1}^{S} f_L(S - NS_0) \cdot \sum_{D_R=NS_0+1}^{\infty} (D_R - NS_0) f_R(D_R)}{\sum_{D_R=1}^{\infty} D_R \cdot f_R(D_R)}$

4. Numerical Experiment

Design of the experiment

The aim of this paper is to assess how approximate fill rate expressions perform when they are used to determine the optimal S that guarantees the achievement of a target fill rate (FR*). In a continuous demand context the determination of base stock levels based on a target service level constrain consists of solving $\delta(FR^*) = S$. However, in a discrete demand context $\delta : \square \mapsto \square$ and therefore the problem turns on finding the minimum stock level that guarantees the achievement of the target fill rate:

$$
\begin{aligned}
&Min\ S \\
&subject\ to \\
&\delta(FR^*) \leq S \quad \forall S \in \square \quad FR^* \in [0,1]
\end{aligned}
$$

To accomplish this purpose, we design a large experiment which consists of calculating the minimum base stock level that guarantees the attainment of the target fill rate using the approximate expressions from Table 1 and the Exact method.

Table 2 presents the set of parameters selected as entry data in this experiment. With regard to the inventory system, an extensive range of values for R, L, FR* are selected in order to consider as many contexts as possible. Regarding demand distributions, we select the most commonly used discrete demand distributions: (1) Bernoulli (θ); (2) binomial (n, θ); (3) Poisson (λ); (4) geometric (θ); and (5) negative binomial (r, θ). Given that the Bernoulli distribution is equivalent to the Binomial distribution with n=1 and the Geometric distribution is equivalent to the Binomial distribution with r=1. Finally our experiment considers Poisson, binomial (Bin) and negative binomial (NegBin) distributions with the appropriate set of parameters. To select parameters of each distribution we focus on considering the four demand categories suggested by Syntetos (2005), i.e. smooth, lumpy, intermittent and erratic. The experiment considers every feasible combination of these values per factor leading to 235,620 different cases.

Table 2. Experimental Factors and Values

Demand distribution			
Poisson(λ)	*λ*	=	0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.75, 0.9, 1, 1.25, 1.5, 1.75, 2, 2.5, 3, 4, 5, 7, 10, 15, 20
Bin(n, θ)	*n*	=	1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 15, 20
	θ	=	0.01, 0.05, 0.1, 0.15, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99
NegBin(r, θ)	*r*	=	0.05, 0.1, 0.2, 0.25, 0.3, 0.4, 0.5, 0.75, 0.9, 1, 1.25, 1.5, 1.75, 2, 2.5, 3, 3.5, 4, 5
	θ	=	0.1, 0.15, 0.25, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99
Inventory system			
Target *fill rate, FR**		=	0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95, 0.99
Review Period, *R*		=	2, 3, 4, 5, 7, 10, 15, 20
Lead time, *L*		=	1, 3, 5, 7, 10, 15, 20

The experiment is designed as follows: (1) we combine a set of input parameters of inventory policy (R and L), demand (distribution and parameters) and target fill rate (FR*); (2) we calculate the minimum S that guarantees the attaining of the FR* with the Exact method; and (3) we calculate the minimum S with approximate methods presented in Section III. The experiment has been programming using JAVA language and will be available upon request.

Analysis of the Results

Given the high number of cases included in the experiment, we carry out an exploratory analysis through the Classification and Regression Trees (CART). This non-parametric analysis is one of the main techniques used in Data Mining to predict membership of cases in the classes of a categorical dependent variable from their measurements on one or more segmentation variables. Therefore its main goal is to predict or explain responses of the categorical dependent variable Breiman (1984). The aim of using CART to analyze our experimental results has the purpose of identifying clusters of cases where a fill rate method shows an homogeneous performance. As a result, the application of CART allows us to identify under which conditions any of the approximate methods is accurate enough to estimate optimal base stock levels.

For every of the 235,620 cases we need to define a categorical dependent variable and a set of independent variables. On one hand, we define a categorical dependent variable ("Best Method") which indicates the simplest and fastest method that leads to the exact base stock level. We use the cyclomatic complexity to compute the simplicity. Note that the Exact method is selected as the best method if none of the approximate methods fulfils the condition. As independent variables of the analysis we select those relating to the simplifications assumed in the derivation of the approximate methods and those relating to the inventory policy, the distribution function or the target fill rate. More concretely we include: the random variable, the squared coefficient of variation of demand sizes (CV2), the average inter-demand interval (p), the target fill rate (FR*), the probability of zero demand (P(0)) and the average demand during R+L (μRL).

The analysis is carried out by using the software STATISTICA (version 8.0). The resulting CART (see Figure 2) has 8 intermediate nodes and 9 final nodes which are explained by four segmentation variables: μRL and CV2, P(0) and FR*. The intermediate nodes help us to understand the structure of the classification scheme, but the final nodes establish the scheme itself. Each node shows: an identity number (ID), the number of cases the node has, the predicted Best method, and the histogram of the cases where, from left to right, Trad, Teunter, Silver, HW, Johnson and Babiloni approximations are observed as Best method. As can be seen, the right branch (when CV2>0.327) of the tree includes a high number of cases (70,686) where the best method is the Exact one. For that reason, we carry out a new CART of this node in order to understand the performance of the approximate expression in those cases. Figure 3 shows the resultant CART.

Figure 2. CART for the best method to compute optimal base stock levels

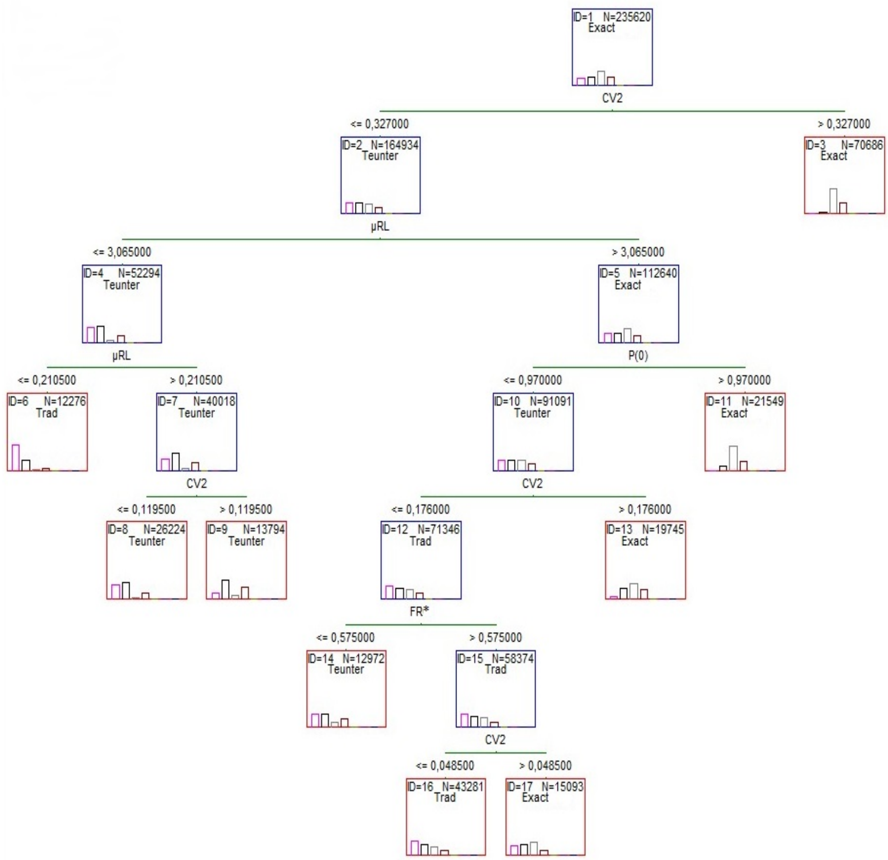
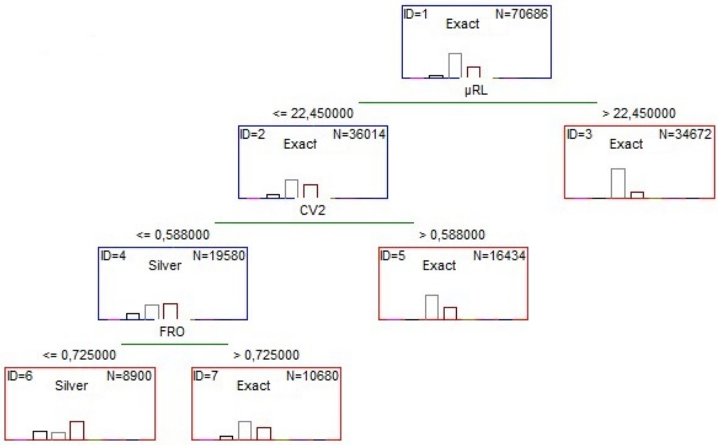


Figure 3. CART for the best method to compute optimal base stock levels in Node 3



For a better understanding and applicability of the results, Figures 4-8 show a graphical representation of the final nodes of CART depending on the FR* and P(0) level. The space of representation is defined by two of the resulted segmentation variables: CV2 and µRL. Note that nodes 6, 8, 9, 3.3 and 3.5 are independent of the FR* and P(0) and therefore they appear in all figures. As can be seen, the Trad method, which is the most commonly used in practical environments, is the best method only when the µRL is very low (µRL<0.21). Conversely, only the Exact expression is the best method when the CV2>0.59 and/or µRL>22.45. In the intermediate situations, the different approximations can provide a good performance depending on the FR* and P(0).

Figure 4. Graphical representation of CART final nodes for P(0)>0.97 and FR*>0.725

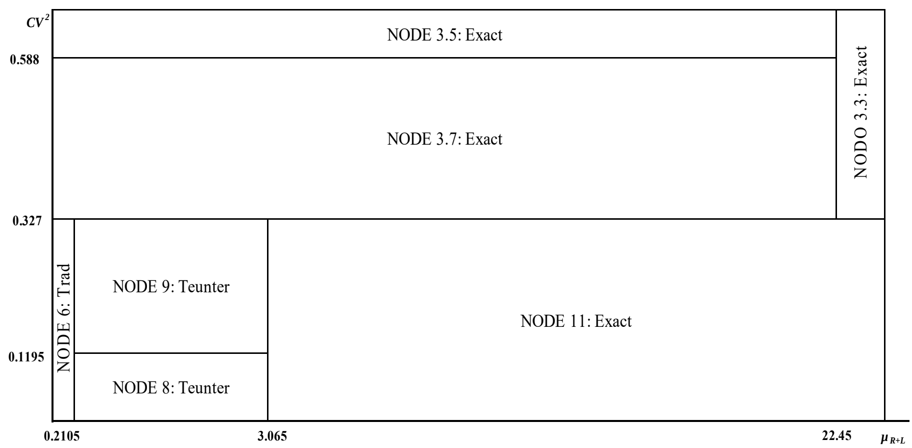


Figure 5. Graphical representation of CART final nodes for P(0)>0.97 and FR*≤0.725

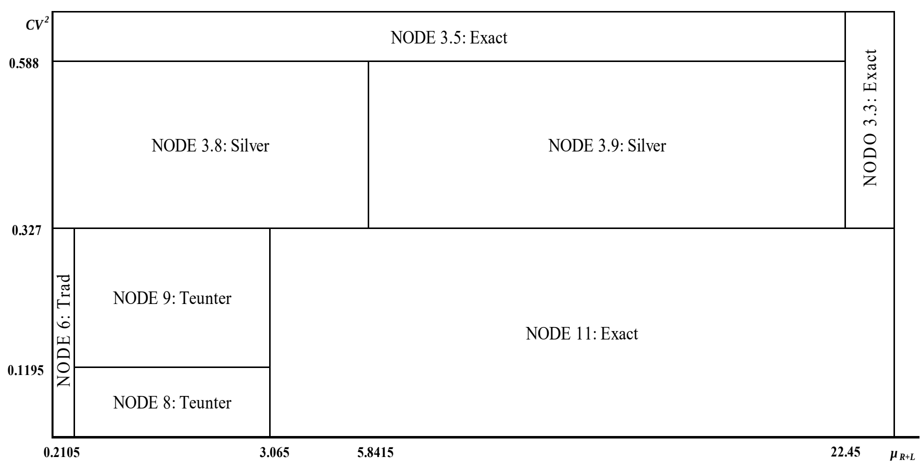


Figure 6. Graphical representation of CART final nodes for P(0)≤0.97 and FR*≤0.575

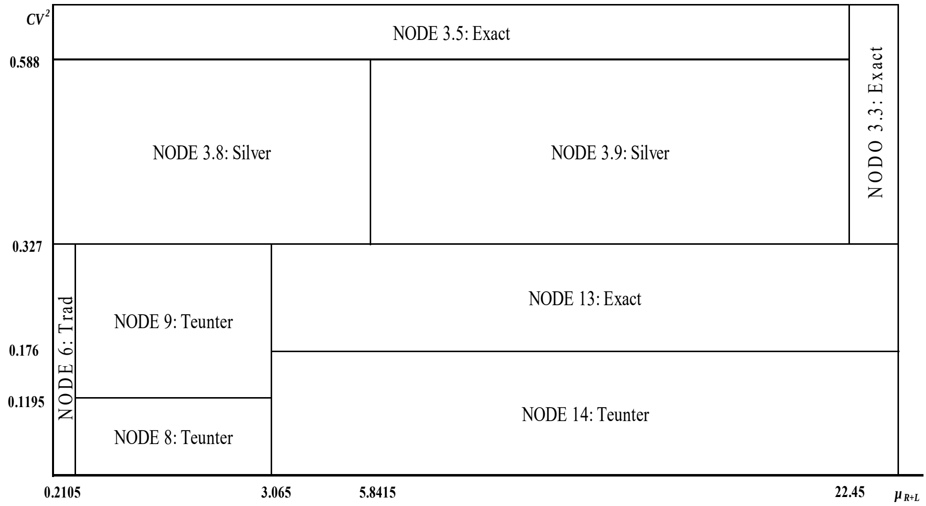


Figure 7. Graphical representation of CART final nodes for P(0)≤0.97 and 0.575<FR*≤0.725

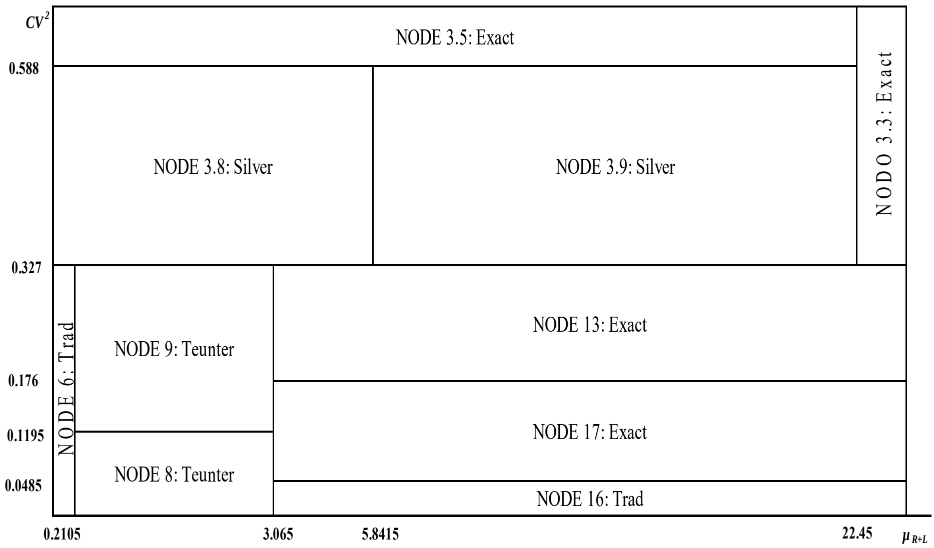
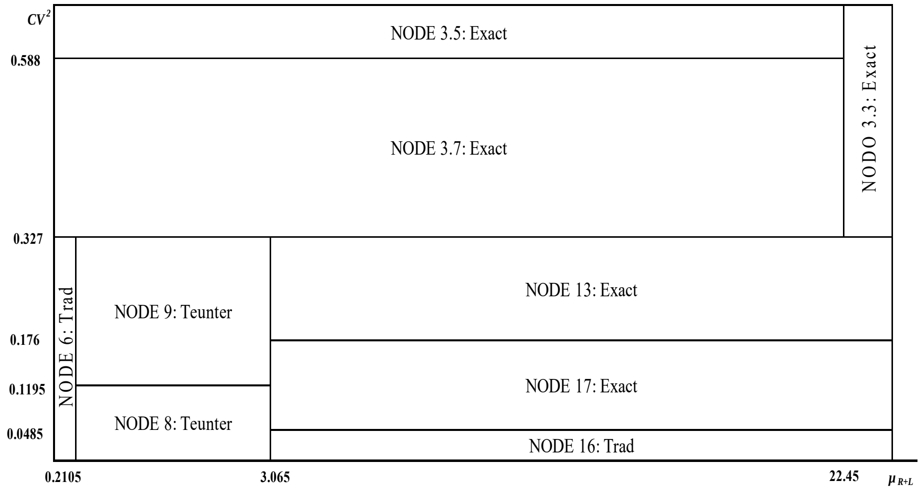


Figure 8. Graphical representation of CART final nodes for P(0)≤0.97 and FR*>0.725



5. Practical Application and Conclusions

A practical tool: a reference framework to select the best method to compute S

The resulted CART reveals the underlying model behind the performance of the approximate methods and identifies zones showing a homogeneous performance of them. However, as a predictive model, it is important to know the classification errors arising from using an approximate method to compute the S in every single node. Furthermore, it is not only important to know the percentage of misclassified cases per node, but also the type of errors that the misclassified cases fall into. In this paper, errors are measured by using the relative error (RE) expressed in terms of per cent: $RE = \left(S_{Exact} - S_{Approx}\right)/S_{Exact}$, where SExact is replaced by the base stock level obtained using the Exact method and SApprox by the base stock level obtained using any of the approximate fill rate expressions. This way to compute errors allows us to know not only the magnitude of the error but also the type of error by analyzing its sign. When the relative error is negative, the approximate method gives a base stock level which is greater than the exact one (SApprox>SExact) and therefore the system reaches the target fill rate, i.e.

there is not impact in the service level of the system, although may cause an increase of the average stock level over the time. This type of classification error is named as CE1 further on. However, if the relative error is positive, the approximate base stock level is lower than the exact one (SApprox < SExact) which means that the system is not reaching the target fill rate which it is designed for. This type of classification error is called CE2 from this point onwards. The impact of having either CE1 or CE2 makes it necessary to clearly identify the type of misclassified cases arising from the use of an approximate method instead of the Exact one in every single node. Table 3 summarizes the misclassified cases per node and categorizes them according to their mean (μ) and standard deviation (σ) which has to be placed in the context of the size of the stock level, S. Note that in Table 3 we include in the last column the results of H&W, Babiloni and Teunter since they compute accurately expressions (2) and (3), and therefore, their results are the same.

The four first columns of Table 3 identify the node, the rest of them present the percentage of cases where each method computes the exact S (%SExact), the percentage of cases in this node that fall into CE1 (%CE1) or into CE2 (%CE2) and the mean (μCE1 and μCE2) and the standard deviation (σCE1 and σCE2) for each approximate method. For example, if an item presents a CV2 = 0.25 and μRL = 0.20 (first row of Table 3) H&W, Babiloni and Teunter are the methods that compute the exact S the greatest number of cases (95.12%), followed by Johnson (91.59% of cases), Silver (85.35% of cases) and Trad (77.13%). If we use H&W, Babiloni and Teunter the expected error is always type EC1 with μE1 = 86.26 and σCE1 = 23.95, whereas if we use, for example, the Johnson expression we can fall into errors or type EC1 (0.90% of the cases) or type EC2 (7.51% cases). It is important to take into account that in this node, the optimal S is very low, for this reason the mean error and the standard deviation is high.

The result of this analysis reveals that when μRL and CV2 are low, all approximate methods present a good performance. However, when both variables are high, the percentage of error is higher. Then, the Exact method should be selected to determine the optimal base stock level in those cases. For example, if the CV2 of an item is 0.5 and μRL is 25 the percentage of cases where approximate methods compute the exact stock level is very low: less than 1.5% for Trad, Johnson, H&W, Babiloni and Teunter and only 18.26% for Silver expression. In this case, the risk of applying approximate methods is very high, so that the Exact expression would be selected as the most efficient method.

Table 3 can be used as a reference framework for determining optimal base stocks in a periodic review policy. This reference framework has important practical implications since it can be used both as a decision and as correcting tool. On one hand, managers can use it to decide the most efficient method to establish the optimal base stock level knowing the characteristics of the item. On the other hand, if a company is already using one of the approximations, the results of this work provide information about the risk derived from using it and managers can calculate the expected relative error and correct it. In practical environments, managers demand easily understood procedures such as spreadsheets or tables Bijvank et al (2012) that would be more used than exact expressions which are hardly implementable in practical information systems. With the reference framework proposed in this paper we fulfil this purpose.

Table 3. Reference Framework to Determine the Optimal Base Stock in a Periodic Review System

NODE				*Trad*				*Silver*							*Johnson*							*HW-Teunter-Babiloni*			
CV2	μ_{RL}	P(0)	FR*	% S_{Exact}	% EC1	μ_{EC1}	σ_{EC1}	% S_{Exact}	% EC1	μ_{EC1}	σ_{EC1}	% EC2	μ_{EC2}	σ_{EC2}	% S_{Exact}	% EC1	μ_{EC1}	σ_{EC1}	% EC2	μ_{EC2}	σ_{EC2}	% S_{Exact}	% EC1	μ_{EC1}	σ_{EC1}
≤ 0.327	≤ 0.2105	--	--	77.13	22.87	-93.55	22.43	85.35	--	--	--	14.65	48.60	5.09	91.59	0.90	-78.56	27.96	7.51	48.55	4.74	95.11	4.89	-86.26	23.95
≤ 0.120	0.2105-3.065	--	--	51.44	48.56	-57.74	31.61	59.29	2.84	-31.02	11.68	37.87	40.74	11.12	43.50	1.35	-40.85	25.57	55.15	49.50	14.28	81.09	18.91	-53.29	29.52
0.120-0.327	0.2105-3.065	--	--	24.26	75.74	-54.95	37.11	52.75	3.84	-27.04	10.88	43.41	37.96	12.43	35.29	3.41	-46.41	28.98	61.29	49.45	16.13	63.26	36.74	-47.75	28.43
≤ 0.327	> 3.065	> 0.97	--	2.97	97.03	-9.46	8.85	32.36	63.72	-4.59	4.58	3.92	11.08	6.33	5.66	9.16	-6.16	6.01	85.18	66.90	38.61	12.09	87.91	-6.32	6.10
0.176-0.327	> 3.065	≤ 0.97	--	17.14	82.86	-11.54	11.17	53.06	40.53	-6.53	6.40	6.41	14.38	6.03	10.81	4.00	-6.94	5.86	85.34	66.59	35.63	37.45	62.55	-8.88	8.09
≤ 0.176	> 3.065	≤ 0.97	≤ 0.575	58.47	41.53	-11.83	12.22	82.12	14.35	-5.08	7.56	3.53	17.29	6.85	9.70	1.38	-7.15	7.99	88.92	89.10	18.31	72.35	27.65	-8.88	10.44
≤ 0.049	> 3.065	≤ 0.97	> 0.575	66.31	33.69	-6.82	7.18	75.78	22.87	-4.86	5.70	1.35	12.99	5.17	32.47	2.68	-4.97	5.04	64.84	75.17	37.02	72.82	27.18	-5.89	6.45
0.049-0.176	> 3.065	≤ 0.97	> 0.575	41.99	58.01	-9.59	8.94	60.99	34.41	-6.62	7.53	4.60	14.01	4.59	17.98	3.39	-4.70	5.52	78.63	64.92	37.68	55.83	44.17	-8.14	8.14
> 0.327	> 22.45	--	--	0.12	99.88	-9.13	9.51	18.26	79.35	-3.73	2.90	2.40	2.70	1.25	1.17	8.94	-4.34	3.61	89.89	70.55	39.88	0.67	99.33	-5.46	4.67
> 0.588	≤ 22.45	--	--	0.00	100.00	-56.92	52.23	32.94	34.46	-11.41	6.01	32.60	22.38	17.02	4.62	18.22	-23.64	23.12	77.15	62.35	31.00	0.71	99.29	-28.70	23.12
0.327-0.588	≤ 22.45		> 0.725	0.56	99.44	-23.23	20.97	39.61	41.25	-10.34	5.84	19.15	23.40	15.41	14.85	11.67	-14.51	12.50	73.48	48.87	33.34	11.72	88.28	-15.56	11.97
0.327-0.588	≤ 22.45	--	≤ 0.725	2.61	97.39	-41.19	39.93	59.33	14.88	-12.90	7.17	25.80	29.86	16.16	11.17	6.85	-27.62	27.29	81.98	73.47	20.66	27.30	72.70	-24.01	22.65

Summary and Conclusions

This paper focuses on how to compute the optimal base stock level of an item or item category given a target fill rate when demand follows any discrete distribution function and the inventory is periodically reviewed. The fill rate is one of the service metric most commonly studied in inventory research. It is defined as the fraction of demand that is immediately fulfilled from the on-hand stock. Despite its definitions is simple, we find different interpretations of it in the literature, and consequently, different expressions to compute it. In a (R, S) inventory system, only [8] propose an exact method of the fill rate in a discrete demand context based explicitly on its definition. However, there are several approximations that simplify its computation through the expected shortage and calculate the fill rate as expression (2) or (3). However [6] point out that approximate fill rate may lead to underestimate the fill rate which has important consequences when the fill rate is used to determine the optimal policy. The underestimation of the fill rate leads the company to an unnecessary increase in the average stock level and thus to an increase in the holding costs of the system. This inefficiency is especially relevant in industries in which the unit cost of the item is high and/or storage space is limited. This paper evaluates under which conditions these approximate expressions present a good performance when they are used to determine the optimal parameters of the inventory policy.

To accomplish the objective of this paper, we carry out a wide experiment (235,620 cases) and we use the classification and regression trees (CART) as exploratory technique to find homogenous regions (nodes) in which approximations have the same performance in the determination of S (see Figure 3 and Figure 2). Those regions are delimitated by segmentation variables (independent variables) that influence the performance of the dependent variable -the simpler approximate method that provides the same S than the Exact does- and determine the underlying model behind the results. For our analysis the segmentation variables are: the squared coefficient of variation of demand sizes, CV2; the mean of the random variable during R+L, μRL; and with less importance for the model, the target FR* and the probability of zero demand, P(0). The segmentations variables CV2and μRL have allowed representing the model into two dimension charts (see Figures 4-8). The new space of representation that arises from the CART analysis is not the same that that suggested by Syntetos (2005) but it is important to remark that both spaces consider the CV2. Note that this new space of representation includes the characteristics of demand pattern due to CV2 or μ but also characteristics of the inventory system as the review period R, or the lead time, L.

To complete the analysis and in order to have a better understanding of the approximations, we compute the percentage of misclassified cases of each zone and the average and standard deviation of their relative errors. The results of this analysis are summarized in Table 3 that can be used as a reference framework to help managers in the decision making process of selecting the best method to estimate the base stock level in a (R, S) inventory system. The proposed reference framework contains all the relevant information for selecting one method to compute base stock levels given a target FR* when considering discrete demand and the inventory system is managed periodically. The practical purpose of this reference framework is as decision tool. In this sense it may assist managers to select the most simple and accurate method to compute the base stock level of an item or item categories. Furthermore Table 3 summaries the risks (type, percentage and average and standard deviation of errors) that arise from using the fill rate approximations when computing base stock levels and thus managers can, in practice, detect and correct them.

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